Einstein's explanation of Specific heat of Solids

Classically, each independent vibrational mode gets **average** energy kT (half due to kinetic energy and half due to potential energy).

If the vibrational modes are quantized, this average energy for the model with frequency ν is $h\nu/(e^{(h\nu/kT)} - 1)$. Hence the internal energy of the solid is

$$U = \frac{3Nh\nu}{e^{(h\nu/kT)} - 1}.$$

For $h\nu/kT \ll 1$ (or for large T), this reduces to the classical formula.

For low temperatures, we have $h\nu/kT \gg 1$ and the internal energy becomes

$$U = 3Nh\nu e^{-(h\nu/kT)},$$

and the specific heat is

$$C = \frac{\partial U}{\partial T} = 3R\left(\frac{h\nu}{kT}\right)^2 e^{-(h\nu/kT)},$$

leading to $C \to 0$ as $T \to 0$.