Suppose that $A_1, ..., A_n$ are Borel sets, that is they belong to ß. Define the following sets: $B_1 = A_1$, and $B_n = A_n \cap (A_1 \cup A_2 \cup ... \cup A_{n-1})^c$, And let S be the universal set.

Show $\bigcup_{i=1}^n A_i = \bigcup_{i=1}^n B_i$.

Obviously $\bigcup_{i=1}^1 A_i = A_1 = \bigcup_{i=1}^1 B_i = B_1$. Now assume the statement is true for $n = k$, so that we have $\bigcup_{i=1}^k A_i = \bigcup_{i=1}^k B_i$.

Then $\bigcup\limits_{i=1}^{k+1} B_i = \left(\bigcup\limits_{i=1}^{k} B_i\right) \cup B_{k+1} = (\bigcup\limits_{i=1}^{k} A_i) \cup B_{k+1}$

$= (\bigcup\limits_{i=1}^{k} A_i) \cup (A_{k+1} \cap (A_1 \cup A_2 \cup ...A_k)^c)$

$= (\bigcup\limits_{i=1}^{k} A_i) \cup (A_{k+1} \cap A_1^c \cap A_2^c \cap ... \cap A_k^c)$

$= \left(\left(\bigcup\limits_{i=1}^{k} A_i\right) \cup (A_1^c \cap A_2^c \cap ... \cap A_k^c)\right) \cap \left(\left(\bigcup\limits_{i=1}^{k} A_i\right) \cup A_{k+1}\right)$

$= S \cap \left(\left(\bigcup\limits_{i=1}^{k} A_i\right) \cup A_{k+1}\right)$

$= \bigcup\limits_{i=1}^{k+1} A_i$