Box Tipping Analysis

This analysis considers a box, strapped to a pallet, as shown in the figure below. The assembly overall dimensions are $b \times h$, and the composite center of mass is located at (u, v) as indicated.

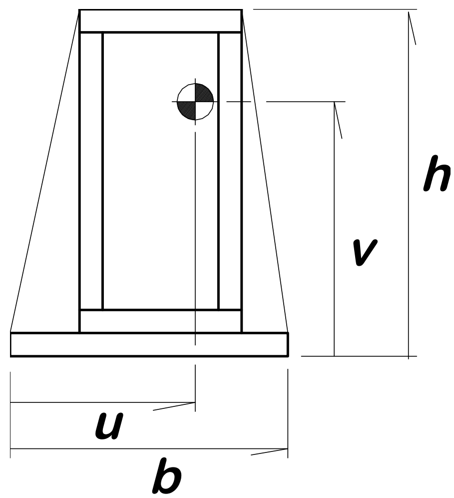

Fig. 1 System Definition

The question is whether the box will roll on over if it is first tipped to the left an angle θ_o and then released. The eccentric location of the center of mass is a key feature of this question.
Four states are considered as shown in Fig. 2 below. The initial condition of the test is the second state shown in this sequence.

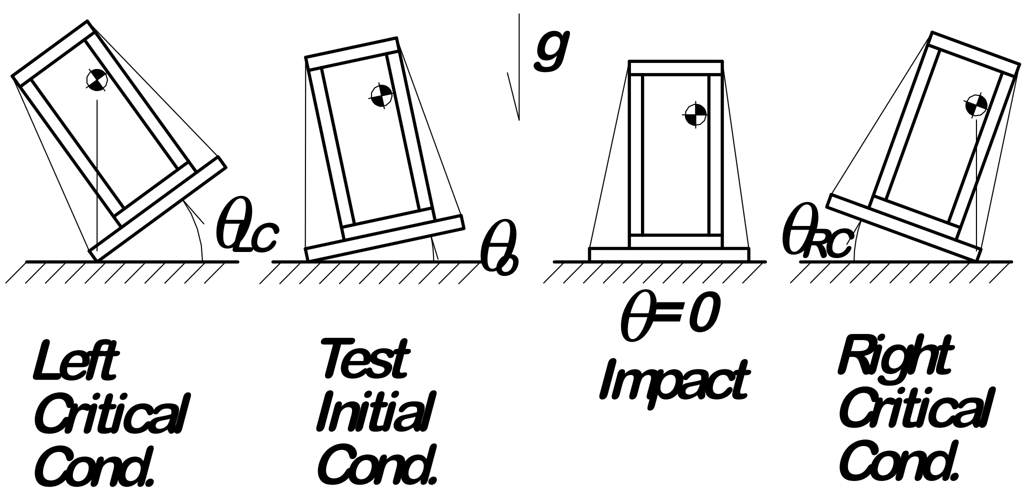

Fig. 2 Four System States

Left Critical Condition

The left critical condition is the limiting position to avoid tipping the box to the left. It is apparent that the initial test condition, $\theta = \theta_o$ must be such that $\theta_o \leq \theta_{LC}$. In the left critical condition, the center of mass is directly over the left edge. Thus

$$\theta_{LC} + \arctan\left(\frac{v}{u}\right) = \frac{\pi}{2}$$

or

$$\theta_{LC} = \frac{\pi}{2} - \arctan\left(\frac{v}{u}\right)$$

Test Initial Condition

In the test initial condition, the box is at rest but the center of mass is elevated. The potential energy of the box is

$$V_{TIC} = Mg\left(u\sin\theta_o + v\cos\theta_o - v\right)$$

with respect to the position $\theta = 0$. The initial kinetic energy of the box is

$$T_{TIC} = 0$$

because the box is at rest.

Impact

When the box is released from rest in the initial condition, it falls to the floor where impact occurs. The impact is a complicated process, involving a distributed impulsive loading on the base of the pallet and acting for a very short, but indeterminate, time interval.

From the initial release down to the instant of impact, energy is conserved. Therefore,

$$T_{TIC} + V_{TIC} = T_{\text{Impact Initial}} + V_{\text{Impact Initial}}$$

Because this position is the reference state for potential energy

$V_{\text{Impact Initial}} = V_{\text{Impact Final}} = 0$. Thus

$$T_{\text{Impact Initial}} = V_{TIC} = Mg\left(u\sin\theta_o + v\cos\theta_o - v\right)$$

Real impacts always involve the loss of some energy, but the amount is difficult to quantify. In the limiting case, energy is conserved in an impact, so that

$$T_{\text{Impact Final}} \leq T_{\text{Impact Initial}} = Mg\left(u\sin\theta_o + v\cos\theta_o - v\right)$$

Right Critical Condition

The right critical condition is reaced when the box is on the verge of tipping to the right. At this condition, the center of mass is directly over the right edge of the pallet as shown in the fourth image of Fig. 2. At this position, the angle θ_{RC} is such that

$$\theta_{RC} + \arctan\left(\frac{v}{b-u}\right) = \frac{\pi}{2}$$

or

$$\theta_{RC} = \frac{\pi}{2} - \arctan\left(\frac{v}{b-u}\right)$$

In the right critical condition, the potential energy of the box is

$$V_{RC} = Mg\left[(b-u)\sin\theta_{RC} + v\cos\theta_{RC} - v\right]$$

and the kinetic energy is

$$T_{RC} \geq 0$$

where the equality represents the limiting condition.

Resolution

The whole question comes down to, "Is the total energy at the end of impact sufficient to reach the right critical condition?" If it is, then the box will tip over; if it is not, the box cannot tip. Thus, to assure that the box does not tip,

$$T_{\text{Impact Final}} < V_{RC}$$
$$Mg\,(u\sin\theta_o + v\cos\theta_o - v) < Mg\,[(b-u)\sin\theta_{RC} + v\cos\theta_{RC} - v]$$
$$u\sin\theta_o + v\cos\theta_o - v < (b-u)\sin\theta_{RC} + v\cos\theta_{RC} - v$$
$$u\sin\theta_o + v\cos\theta_o < (b-u)\sin\theta_{RC} + v\cos\theta_{RC}$$

As long as this relation is satisfied, the box will not tip, even with energy conserved in the impact process. For specified values of b, u, v it is possible to solve this relation for the maximum value of θ_o, the most extreme test condition that will not tip the box. Conversely, for given dimensions and θ_o, it is possible to test whether or not the box tips.