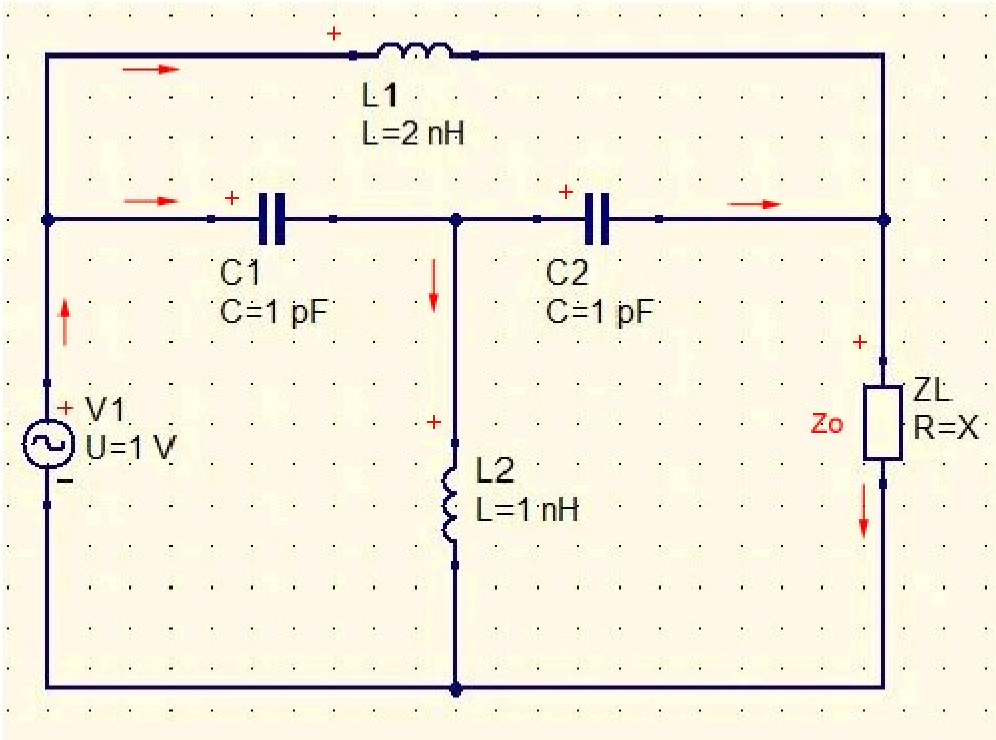

First I'll rename ZL = Zo for clarity later

KCL: $Iv1 - Ic1 - Il1 = 0$ KVL: $-V1 + Vc1 + Vl2 = 0$ ---> $Ic1 \cdot Zc1 + Il2 \cdot Zl2 = V1$

$Ic1 - Ic2 - Il2 = 0$ $-Vl2 + Vc2 + Vzl = 0$ $-Il2 \cdot Zl2 + Ic2 \cdot Zc2 + Io \cdot Zo = 0$

$Il1 + Ic2 - Io = 0$ $Vl1 - Vc2 - Vc1 = 0$ $Il1 \cdot Zl1 - Ic2 \cdot Zc2 - Ic1 \cdot Zc1 = 0$

$Io + Il2 - Iv1 = 0$ $-V1 + Vl1 + Vzl = 0$ $Il1 \cdot Zl1 + Io \cdot Zo = V1$

the last line are redundant equations (the ground node and the outer loop).

$$\begin{pmatrix} 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \\ 0 & Zc1 & 0 & Zl2 & 0 & 0 \\ 0 & 0 & 0 & -Zl2 & Zc2 & Zo \\ 0 & -Zc1 & Zl1 & 0 & -Zc2 & 0 \end{pmatrix} \cdot \begin{pmatrix} Iv1 \\ Ic1 \\ Il1 \\ Il2 \\ Ic2 \\ Io \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ V1 \\ 0 \\ 0 \end{pmatrix} \quad ==> \quad \begin{pmatrix} Iv1 \\ Ic1 \\ Il1 \\ Il2 \\ Ic2 \\ Io \end{pmatrix} = \begin{pmatrix} 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \\ 0 & Zc1 & 0 & Zl2 & 0 & 0 \\ 0 & 0 & 0 & -Zl2 & Zc2 & Zo \\ 0 & -Zc1 & Zl1 & 0 & -Zc2 & 0 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ V1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} Iv1 \\ Ic1 \\ Il1 \\ Il2 \\ Ic2 \\ Io \end{pmatrix} = \frac{V1 \cdot \begin{bmatrix} (Zo \cdot Zc1 + Zo \cdot Zc2 + Zo \cdot Zl1 + Zc1 \cdot Zc2 + Zc1 \cdot Zl2 + Zc2 \cdot Zl1 + Zc2 \cdot Zl2 + Zl1 \cdot Zl2) \\ (Zo \cdot Zc2 + Zo \cdot Zl1 + Zc2 \cdot Zl1 + Zl1 \cdot Zl2) \\ (Zo \cdot Zc1 + Zc1 \cdot Zc2 + Zc1 \cdot Zl2 + Zc2 \cdot Zl2) \\ (Zo \cdot Zc1 + Zo \cdot Zc2 + Zo \cdot Zl1 + Zc2 \cdot Zl1) \\ -(Zo \cdot Zc1 - Zl1 \cdot Zl2) \\ (Zc1 \cdot Zc2 + Zc1 \cdot Zl2 + Zc2 \cdot Zl2 + Zl1 \cdot Zl2) \end{bmatrix}}{(Zo \cdot Zc1 \cdot Zc2 + Zo \cdot Zc1 \cdot Zl1 + Zo \cdot Zc1 \cdot Zl2 + Zo \cdot Zc2 \cdot Zl2 + Zo \cdot Zl1 \cdot Zl2 + Zc1 \cdot Zc2 \cdot Zl1 + Zc1 \cdot Zl1 \cdot Zl2 + Zc2 \cdot Zl1 \cdot Zl2)}$$

Now add the symmetry conditions: $Zc2 = Zc1 = Zc$ $\quad Zl1 = 2\cdot Zl$ $\quad Zl2 = Zl$

$$\begin{pmatrix} Iv1 \\ Ic1 \\ Il1 \\ Il2 \\ Ic2 \\ Io \end{pmatrix} = \begin{pmatrix} 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \\ 0 & Zc & 0 & Zl & 0 & 0 \\ 0 & 0 & 0 & -Zl & Zc & Zo \\ 0 & -Zc & 2\cdot Zl & 0 & -Zc & 0 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ V1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} Iv1 \\ Ic1 \\ Il1 \\ Il2 \\ Ic2 \\ Io \end{pmatrix} = \frac{V1}{\left(2\cdot Zc^2\cdot Zl + Zo\cdot Zc^2 + 4\cdot Zc\cdot Zl^2 + 4\cdot Zo\cdot Zc\cdot Zl + 2\cdot Zo\cdot Zl^2\right)} \cdot \begin{bmatrix} Zc^2 + 4\cdot Zc\cdot Zl + 2\cdot Zo\cdot Zc + 2\cdot Zl^2 + 2\cdot Zo\cdot Zl \\ 2\cdot Zl^2 + 2\cdot Zc\cdot Zl + Zc\cdot Zo + 2\cdot Zl\cdot Zo \\ Zc\cdot(Zc + 2\cdot Zl + Zo) \\ 2\cdot Zc\cdot Zl + 2\cdot Zc\cdot Zo + 2\cdot Zl\cdot Zo \\ 2\cdot Zl^2 - Zc\cdot Zo \\ Zc^2 + 2\cdot Zc\cdot Zl + 2\cdot Zl^2 \end{bmatrix}$$

or, collecting Zo terms:

$$\begin{pmatrix} Iv1 \\ Ic1 \\ Il1 \\ Il2 \\ Ic2 \\ Io \end{pmatrix} = \frac{V1}{\left[2\cdot Zc\cdot Zl\cdot(Zc + 2\cdot Zl) + \left(Zc^2 + 4\cdot Zc\cdot Zl + 2\cdot Zl^2\right)\cdot Zo\right]} \cdot \begin{bmatrix} Zc^2 + 4\cdot Zc\cdot Zl + 2\cdot Zl^2 + 2\cdot(Zc + Zl)\cdot Zo \\ 2\cdot Zl\cdot(Zc + Zl) + (Zc + 2\cdot Zl)\cdot Zo \\ Zc\cdot(Zc + 2\cdot Zl) + Zc\cdot Zo \\ 2\cdot Zc\cdot Zl + 2\cdot(Zc + Zl)\cdot Zo \\ 2\cdot Zl^2 - Zc\cdot Zo \\ Zc^2 + 2\cdot Zc\cdot Zl + 2\cdot Zl^2 \end{bmatrix}$$

---

when Il2=0 $\quad 2\cdot Zc\cdot Zl + 2\cdot Zc\cdot Zo + 2\cdot Zl\cdot Zo = 0 \quad ==> \quad Zo = -\frac{Zc\cdot Zl}{Zc + Zl}$

Substitute: $\quad Zc = \frac{1}{j\cdot\omega\cdot C1} \quad Zl = j\cdot\omega\cdot L2 \quad ==> \quad Zo = \frac{j\cdot\omega\cdot L2}{C1\cdot L2\cdot\omega^2 - 1}$

$$\begin{pmatrix} Iv1 \\ Ic1 \\ Il1 \\ Il2 \\ Ic2 \\ Io \end{pmatrix} = V1\cdot\left(\frac{Zc + Zl}{Zc\cdot Zl}\right)\cdot\begin{pmatrix} 1 \\ \frac{Zl}{Zc + Zl} \\ \frac{Zc}{Zc + Zl} \\ 0 \\ \frac{Zl}{Zc + Zl} \\ 1 \end{pmatrix} = V1\cdot\begin{pmatrix} \frac{1}{Zc} + \frac{1}{Zl} \\ \frac{1}{Zc} \\ \frac{1}{Zl} \\ 0 \\ \frac{1}{Zc} \\ \frac{1}{Zc} + \frac{1}{Zl} \end{pmatrix} = V1\cdot\begin{bmatrix} \left[\frac{\left(1 - C1\cdot L2\cdot\omega^2\right)}{j\,\omega\cdot L2}\right] \\ (j\cdot\omega\cdot C1) \\ (j\cdot\omega\cdot L2) \\ 0 \\ (j\cdot\omega\cdot C1) \\ \left[\frac{\left(1 - C1\cdot L2\cdot\omega^2\right)}{j\,\omega\cdot L2}\right] \end{bmatrix}$$

Now, the easy way:

Since $Vl2 = 0 \implies Il2 = 0$ and $Ic1 = Ic2 = \frac{(V1 - Vl2)}{Zc1} = \frac{V1}{Zc1}$

$$Vc2 = Ic2 \cdot Zc2 = V1 \cdot \left(\frac{Zc2}{Zc1}\right) \implies Vo = -Vc2 = -V1 \cdot \left(\frac{Zc2}{Zc1}\right)$$

$$Il1 = \frac{(V1 - Vo)}{Zl1} = V1 \cdot \frac{\left(1 + \frac{Zc2}{Zc1}\right)}{Zl1} \quad \text{and} \quad Io = Ic2 + Il1 = V1 \cdot \left[\frac{\left(1 + \frac{Zc2}{Zc1}\right)}{Zl1} + \frac{1}{Zc1}\right]$$

$$Zo = \frac{Vo}{Io} = \frac{-\left(\frac{Zc2}{Zc1}\right)}{\left[\frac{\left(1 + \frac{Zc2}{Zc1}\right)}{Zl1} + \frac{1}{Zc1}\right]} = \frac{-Zc2 \cdot Zl1}{Zc1 + Zc2 + Zl1}$$

when $Zc1 = Zc2$ and $Zl1 = 2 \cdot Zl2$ then $Vo = -V1$ and $Zo = \frac{-1}{\frac{1}{Zl2} + \frac{1}{Zc1}} = \frac{-Zl2 \cdot Zc1}{(Zl2 + Zc1)} = \frac{j \cdot \omega \cdot L2}{\left(C1 \cdot L2 \cdot \omega^2 - 1\right)}$

note that this is also $Zo = \frac{-1}{2 \cdot \left[\frac{1}{Zl1} + \frac{1}{(Zc1 + Zc2)}\right]}$

Find the Z parameters of the network, less source and load:

Define: $Cp = \frac{C1 \cdot C2}{(C1 + C2)}$ $\qquad s = 1j \cdot \omega$

$Z11 = \frac{V1}{I1}$ when $I2 = Io = 0$ $\qquad Z11 = \frac{(Zl1 + Zc2) \cdot Zc1}{(Zl1 + Zc2 + Zc1)} + Zl2 = \frac{\left(s \cdot L1 + \frac{1}{s \cdot C2}\right) \cdot \frac{1}{s \cdot C1}}{s \cdot L1 + \frac{1}{s \cdot C1} + \frac{1}{s \cdot C2}} + s \cdot L2$

$$Z11 = \frac{C2 \cdot L1 \cdot s^2 + 1}{s \cdot \left(C1 \cdot C2 \cdot L1 \cdot s^2 + C1 + C2\right)} + s \cdot L2 = \frac{C2 \cdot L1 \cdot s^2 + 1}{s \cdot (C1 + C2) \cdot \left(Cp \cdot L1 \cdot s^2 + 1\right)} + s \cdot L2 = \frac{C1 \cdot C2 \cdot L1 \cdot L2 \cdot s^4 + [(C1 + C2) \cdot L2 + C2 \cdot L1] \cdot s^2 + 1}{s \cdot (C1 + C2) \cdot \left(Cp \cdot L1 \cdot s^2 + 1\right)}$$

$Z22 = \frac{V2}{I2} = -\frac{Vo}{Io}$ when $I1 = 0$ $\qquad$ by symmetry, this is Z11 with C1 and C2 swapped

$$Z22 = \frac{(Zl1 + Zc1) \cdot Zc2}{(Zl1 + Zc1 + Zc2)} + Zl2 = \frac{C1 \cdot C2 \cdot L1 \cdot L2 \cdot s^4 + [(C1 + C2) \cdot L2 + C1 \cdot L1] \cdot s^2 + 1}{s \cdot (C1 + C2) \cdot \left(Cp \cdot L1 \cdot s^2 + 1\right)}$$

$Z12 = \frac{V1}{I2} = -\frac{V1}{Io}$ when $I1 = 0$ $\qquad Vo = -Io \cdot Z22$ and $Vl2 = -Io \cdot Zl2$ ==> $Vc2 = Vl2 - Vo = Io \cdot (Z22 - Zl2)$

$$Ic1 = -Il1 = \frac{-Vc2}{(Zl1 + Zc1)} = Io \cdot \frac{(Zl2 - Z22)}{(Zl1 + Zc1)}$$

then $V1 = Vc1 + Vl2 = Zc1 \cdot Ic1 - Io \cdot Zl2 = Io \cdot \left[Zc1 \cdot \frac{(Zl2 - Z22)}{(Zl1 + Zc1)} - Zl2\right] = -Io \cdot \left(\frac{Z22 \cdot Zc1 + Zl1 \cdot Zl2}{Zc1 + Zl1}\right)$

$$Z12 = \frac{Z22 \cdot Zc1 + Zl1 \cdot Zl2}{Zc1 + Zl1} = \frac{\left[\frac{(Zl1 + Zc1) \cdot Zc2}{(Zl1 + Zc1 + Zc2)} + Zl2\right] \cdot Zc1 + Zl1 \cdot Zl2}{Zc1 + Zl1} = \frac{Zc1 \cdot Zc2 + (Zc1 + Zc2) \cdot Zl2 + Zl1 \cdot Zl2}{Zc1 + Zc2 + Zl1}$$

$$Z12 = \frac{C1 \cdot C2 \cdot L1 \cdot L2 \cdot s^4 + (C1 + C2) \cdot L2 \cdot s^2 + 1}{s \cdot (C1 + C2) \cdot \left(Cp \cdot L1 \cdot s^2 + 1\right)}$$

$Z21 = \frac{V2}{I1} = \frac{Vo}{I1}$ when $I1 = 0$ $\qquad$ by symmetry, this is Z11 with C1 and C2 swapped, which yields $\qquad Z21 = Z12$

Now add the symmetry conditions: $\qquad C1 = C2 = C \qquad L2 = L \qquad L1 = 2 \cdot L$ ==> $Cp = \frac{C}{2}$

$$Z11 = Z22 = \frac{2 \cdot C^2 \cdot L^2 \cdot s^4 + 4 \cdot C \cdot L \cdot s^2 + 1}{2 \cdot C \cdot s \cdot \left(C \cdot L \cdot s^2 + 1\right)} \qquad Z12 = Z21 = \frac{2 \cdot C^2 \cdot L^2 \cdot s^4 + 2 \cdot C \cdot L \cdot s^2 + 1}{2 \cdot C \cdot s \cdot \left(C \cdot L \cdot s^2 + 1\right)}$$

Forward voltage gain with load Z2 in Z-parameters, the load gives the relation: $V2 = -I2 \cdot Z2$

The Z-parameters give $$V = \begin{pmatrix} V1 \\ V2 \end{pmatrix} = Z \cdot I = \begin{pmatrix} Z11 & Z12 \\ Z21 & Z22 \end{pmatrix} \cdot \begin{pmatrix} I1 \\ I2 \end{pmatrix}$$

Substituting for I2 gives $$\begin{pmatrix} V1 \\ V2 \end{pmatrix} = \begin{pmatrix} Z11 & Z12 \\ Z21 & Z22 \end{pmatrix} \cdot \begin{pmatrix} I1 \\ \dfrac{-V2}{Z2} \end{pmatrix}$$ or $$V1 = Z11 \cdot I1 - \left(\frac{Z12}{Z2}\right) \cdot V2$$ $$V2 = Z21 \cdot I1 - \left(\frac{Z22}{Z2}\right) \cdot V2$$

==> $$Z2 \cdot V1 + Z12 \cdot V2 = Z11 \cdot I1$$ $$Z2 \cdot V2 + Z22 \cdot V2 = Z21 \cdot I1$$ ==> $$Z21 \cdot (Z2 \cdot V1 + Z12 \cdot V2) - Z11 \cdot (Z2 \cdot V2 + Z22 \cdot V2) = 0$$

==> $$\frac{V2}{V1} = \frac{Z2 \cdot Z21}{Z11 \cdot (Z2 + Z22) - Z12 \cdot Z21}$$ by symmetry $$Z11 = Z22$$ $$Z12 = Z21$$ $$\frac{V2}{V1} = \frac{Z2 \cdot Z12}{Z11^2 + Z2 \cdot Z11 - Z12^2}$$

$$\frac{V2}{V1} = \frac{\left(2 \cdot C^2 \cdot L^2 \cdot s^4 + 2 \cdot C \cdot L \cdot s^2 + 1\right)}{2 \cdot C^2 \cdot L^2 \cdot s^4 + \dfrac{4 \cdot C \cdot L^2}{Z2} \cdot s^3 + 4 \cdot C \cdot L \cdot s^2 + \dfrac{2 \cdot L}{Z2} \cdot s + 1}$$

Define $$\omega 0 = \frac{1}{\sqrt{\sqrt{2} \cdot L \cdot C}}$$ $$\alpha = \frac{s}{\omega 0}$$ ==> $$\frac{V2}{V1} = \frac{\left(\alpha^4 + \sqrt{2} \cdot \alpha^2 + 1\right)}{\alpha^4 + \sqrt{2} \cdot \left(\dfrac{\sqrt{\sqrt{8}}}{Z2} \cdot \sqrt{\dfrac{L}{C}}\right) \cdot \alpha^3 + 2 \cdot \sqrt{2} \cdot \alpha^2 + \left(\dfrac{\sqrt{\sqrt{8}}}{Z2} \cdot \sqrt{\dfrac{L}{C}}\right) \cdot \alpha + 1}$$

$C := 1\cdot10^{-12}\cdot F \qquad L := 1\cdot10^{-9}\cdot H \qquad Z2 := 50\cdot\Omega$

$$H(s) := \frac{2\cdot C^2\cdot L^2\cdot s^4 + 2\cdot C\cdot L\cdot s^2 + 1}{2\cdot C^2\cdot L^2\cdot s^4 + \frac{4\cdot C\cdot L^2}{Z2}\cdot s^3 + 4\cdot C\cdot L\cdot s^2 + \frac{2\cdot L}{Z2}\cdot s + 1} \qquad H\left(j\cdot\frac{1}{\sqrt{L\cdot C}}\right) = -0.385 + 0.487i$$

$fmin := 10^8 \qquad fmax := 10^{11} \qquad P := 5000 \quad p := 0..P-1 \qquad f_p := fmin\cdot10^{\frac{p}{P-1}\cdot\log\left(\frac{fmax}{fmin}\right)} \qquad s_p := 2\cdot\pi\cdot j\cdot f_p\cdot Hz$

$Mag_p := 20\cdot\log\left(\left|H\left(s_p\right)\right|\right) \qquad Ph_p := \frac{180}{\pi}\cdot\arg\left(H\left(s_p\right)\right) \qquad \max(Mag) = 6.473 \qquad \min(Ph) = -179.718$

$\min(Mag) = -4.543$