Extended Steinhart Heart's polynom (1) is just a polynom.

$$\frac{1}{T} = A + B\ln(R) + C \cdot (\ln(R))^2 + D \cdot (\ln(R))^3 \tag{1}$$

We can calculate it's coefficents A, B, C and D very elegantly. Because there are four coeficcients, we will need to create a system of four equations like this:

$$\begin{aligned}
\frac{1}{T_1} &= A + B\ln(R_1) + C \cdot (\ln(R_1))^2 + D \cdot (\ln(R_1))^3 \\
\frac{1}{T_2} &= A + B\ln(R_2) + C \cdot (\ln(R_2))^2 + D \cdot (\ln(R_2))^3 \\
\frac{1}{T_3} &= A + B\ln(R_3) + C \cdot (\ln(R_3))^2 + D \cdot (\ln(R_3))^3 \\
\frac{1}{T_4} &= A + B\ln(R_4) + C \cdot (\ln(R_4))^2 + D \cdot (\ln(R_4))^3
\end{aligned}$$

Now we can look inside any documentation for a well known NTC thermistor e.g. ./documentation–ntc.pdf where we find a table consisting of resistance measurments[1] at known temperatures. From that table we take four values which should be as far appart as possible.

$$\begin{aligned}
R_1 &= 329518\ \Omega & T_1 &= -50\ ^\circ\mathrm{C} \\
R_2 &= 19488\ \Omega & T_2 &= 8\ ^\circ\mathrm{C} \\
R_3 &= 2511\ \Omega & T_3 &= 66\ ^\circ\mathrm{C} \\
R_4 &= 503\ \Omega & T_4 &= 125\ ^\circ\mathrm{C}
\end{aligned}$$

Now we know the values for resistance and temperature and can calculate the coefficients A, B, C and D. Before solving the system we first make it easier on ourselves and execute some substitutions[2].

[1] We look at nominal values while ignoring minimum and maximum values.
[2] Right before the end we will revert these substitutions and calculate coefficients.

$$
\begin{aligned}
\frac{1}{T_1} &= a_1 & \ln(R_1) &= b_1 & (\ln(R_1))^2 &= c_1 & (\ln(R_1))^3 &= d_1 \\
\frac{1}{T_2} &= a_2 & \ln(R_2) &= b_2 & (\ln(R_2))^2 &= c_2 & (\ln(R_2))^3 &= d_2 \\
\frac{1}{T_3} &= a_3 & \ln(R_3) &= b_3 & (\ln(R_3))^2 &= c_3 & (\ln(R_3))^3 &= d_3 \\
\frac{1}{T_4} &= a_4 & \ln(R_4) &= b_4 & (\ln(R_4))^2 &= c_4 & (\ln(R_4))^3 &= d_4
\end{aligned}
$$

After the substitutions the system of equations becomes easier to read.

$$
\begin{aligned}
a_1 &= A + Bb_1 + Cc_1 + Dd_1 \\
a_2 &= A + Bb_2 + Cc_2 + Dd_2 \\
a_3 &= A + Bb_3 + Cc_3 + Dd_3 \\
a_4 &= A + Bb_4 + Cc_4 + Dd_4
\end{aligned} \tag{2}
$$

Now we start solving the system for A. First we express A from (2).

$$
\begin{aligned}
A &= a_1 - Bb_1 - Cc_1 - Dd_1 \\
a_2 &= A + Bb_2 + Cc_2 + Dd_2 \\
a_3 &= A + Bb_3 + Cc_3 + Dd_3 \\
a_4 &= A + Bb_4 + Cc_4 + Dd_4
\end{aligned} \tag{3}
$$

Then we continue by inserting (3) in all the rest.

$$
\begin{aligned}
a_2 &= a_1 - Bb_1 - Cc_1 - Dd_1 + Bb_2 + Cc_2 + Dd_2 \\
a_3 &= a_1 - Bb_1 - Cc_1 - Dd_1 + Bb_3 + Cc_3 + Dd_3 \\
a_4 &= a_1 - Bb_1 - Cc_1 - Dd_1 + Bb_4 + Cc_4 + Dd_4
\end{aligned}
$$

We use the brackets to shorten the system of equations and put all the parts without a coefficient to the left.

$$
\begin{aligned}
a_2 - a_1 &= B(b_2 - b_1) + C(c_2 - c_1) + D(d_2 - d_1) \\
a_3 - a_1 &= B(b_3 - b_1) + C(c_3 - c_1) + D(d_3 - d_1) \\
a_4 - a_1 &= B(b_4 - b_1) + C(c_4 - c_1) + D(d_4 - d_1)
\end{aligned}
\tag{4}
$$

We now extract B from (4).

$$
\begin{aligned}
B &= \frac{(a_2 - a_1) - C(c_2 - c_1) - D(d_2 - d_1)}{(b_2 - b_1)} \\
a_3 - a_1 &= B(b_3 - b_1) + C(c_3 - c_1) + D(d_3 - d_1) \\
a_4 - a_1 &= B(b_4 - b_1) + C(c_4 - c_1) + D(d_4 - d_1)
\end{aligned}
\tag{5}
$$

We insert (5) in all the rest.

$$
a_3 - a_1 = \left[\frac{(a_2 - a_1) - C(c_2 - c_1) - D(d_2 - d_1)}{(b_2 - b_1)}\right](b_3 - b_1) + C(c_3 - c_1) + D(d_3 - d_1) \tag{6}
$$

$$
a_4 - a_1 = \left[\frac{(a_2 - a_1) - C(c_2 - c_1) - D(d_2 - d_1)}{(b_2 - b_1)}\right](b_4 - b_1) + C(c_4 - c_1) + D(d_4 - d_1) \tag{7}
$$

Now we get rid of the fractions, so we multiply (6) and (7) by $(b_2 - b_1)$.

$$\begin{aligned}(a_3-a_1)\cdot(b_2-b_1) = {} & [(a_2-a_1)-C(c_2-c_1)-D(d_2-d_1)]\,(b_3-b_1)+C(c_3-c_1)\cdot(b_2-b_1)\\ & +D(d_3-d_1)\cdot(b_2-b_1)\\ (a_4-a_1)\cdot(b_2-b_1) = {} & [(a_2-a_1)-C(c_2-c_1)-D(d_2-d_1)]\,(b_4-b_1)+C(c_4-c_1)\cdot(b_2-b_1)\\ & +D(d_4-d_1)\cdot(b_2-b_1)\end{aligned}$$

We get rid of the square brackets.

$$\begin{aligned}(a_3-a_1)\cdot(b_2-b_1) &= (a_2-a_1)\cdot(b_3-b_1)-C(c_2-c_1)\cdot(b_3-b_1)-D(d_2-d_1)\cdot(b_3-b_1)\\ &\quad +C(c_3-c_1)\cdot(b_2-b_1)+D(d_3-d_1)\cdot(b_2-b_1)\\ (a_4-a_1)\cdot(b_2-b_1) &= (a_2-a_1)\cdot(b_4-b_1)-C(c_2-c_1)\cdot(b_4-b_1)-D(d_2-d_1)\cdot(b_4-b_1)\\ &\quad +C(c_4-c_1)\cdot(b_2-b_1)+D(d_4-d_1)\cdot(b_2-b_1)\end{aligned}$$

Now we remove the rest of the brackets by multiplying them.

$$\begin{aligned}a_3b_2-a_1b_2-a_3b_1+a_1b_1 &= a_2b_3-a_1b_3-a_2b_1+a_1b_1\\ &\quad -Cc_2b_3+Cc_1b_3+Cc_2b_1-Cc_1b_1-Dd_2b_3+Dd_1b_3+Dd_2b_1-Dd_1b_1\\ &\quad +Cc_3b_2-Cc_1b_2-Cc_3b_1+Cc_1b_1+Dd_3b_2-Dd_1b_2-Dd_3b_1+Dd_1b_1\\ a_4b_2-a_4b_1-a_1b_2+a_1b_1 &= a_2b_4-a_1b_4-a_2b_1+a_1b_1\\ &\quad -Cc_2b_4+Cc_1b_4+Cc_2b_1-Cc_1b_1-Dd_2b_4+Dd_1b_4+Dd_2b_1-Dd_1b_1\\ &\quad +Cc_4b_2-Cc_1b_2-Cc_4b_1+Cc_1b_1+Dd_4b_2-Dd_1b_2-Dd_4b_1+Dd_1b_1\end{aligned}$$

We cancel as much as we can.

$$
\begin{aligned}
a_3b_2 - a_1b_2 - a_3b_1 = {} & a_2b_3 - a_1b_3 - a_2b_1 \\
& - Cc_2b_3 + Cc_1b_3 + Cc_2b_1 - Dd_2b_3 + Dd_1b_3 + Dd_2b_1 \\
& + Cc_3b_2 - Cc_1b_2 - Cc_3b_1 + Dd_3b_2 - Dd_1b_2 - Dd_3b_1 \qquad (8) \\
a_4b_2 - a_4b_1 - a_1b_2 = {} & a_2b_4 - a_1b_4 - a_2b_1 \\
& - Cc_2b_4 + Cc_1b_4 + Cc_2b_1 - Dd_2b_4 + Dd_1b_4 + Dd_2b_1 \\
& + Cc_4b_2 - Cc_1b_2 - Cc_4b_1 + Dd_4b_2 - Dd_1b_2 - Dd_4b_1 \qquad (9)
\end{aligned}
$$

In (8) and (9) we move all the parts containing C to the left. We move all the rest to the right.

$$
\begin{aligned}
Cc_2b_3 - Cc_1b_3 - Cc_2b_1 - Cc_3b_2 + Cc_1b_2 + Cc_3b_1 = {} & a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1 \\
& - Dd_2b_3 + Dd_1b_3 + Dd_2b_1 \\
& + Dd_3b_2 - Dd_1b_2 - Dd_3b_1 \qquad (10) \\
Cc_2b_4 - Cc_1b_4 - Cc_2b_1 - Cc_4b_2 + Cc_1b_2 + Cc_4b_1 = {} & a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2 \\
& - Dd_2b_4 + Dd_1b_4 + Dd_2b_1 \\
& + Dd_4b_2 - Dd_1b_2 - Dd_4b_1 \qquad (11)
\end{aligned}
$$

In (10) and (11) we express C on the left side and D on the right side.

$$
\begin{aligned}
C(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1) = {} & a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1 \\
& + D(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1) \qquad (12) \\
C(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1) = {} & a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2 \\
& + D(d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1) \qquad (13)
\end{aligned}
$$

We extract C from (12) and (13).

$$C = \frac{(a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1)}{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)} + D\frac{(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)}{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)} \tag{14}$$

$$C = \frac{(a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2)}{(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)} + D\frac{(d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1)}{(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)} \tag{15}$$

We take C from (14) and insert it in (15) to get a single equation.

$$\begin{gathered}\frac{(a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1)}{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)} + D\frac{(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)}{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)} = \\ \frac{(a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2)}{(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)} + D\frac{(d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1)}{(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)}\end{gathered} \tag{16}$$

We put all the elements containing D on the left and use brackets to only get one D.

$$\begin{gathered}D\left[\frac{(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)}{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)} - \frac{(d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1)}{(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)}\right] = \\ \frac{(a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2)}{(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)} - \frac{(a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1)}{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)}\end{gathered}$$

Now we extract D.

$$D = \frac{\dfrac{(a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2)}{(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)} - \dfrac{(a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1)}{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)}}{\dfrac{(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)}{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)} - \dfrac{(d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1)}{(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)}}$$

We see some common large parts in the denominators of the nested fractions that we could cancel out. This is why we set the common denominator on the nested fractions.

$$D = \frac{\frac{(a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2)\cdot(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1) - (a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1)\cdot(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)}{(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)\cdot(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)}}{\frac{(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)\cdot(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1) - (d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1)\cdot(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)}{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)\cdot(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)}}$$

Now the common denominators of the both nested fractions are the same! and they cancel each other out!

$$D = \frac{(a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2)\cdot(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1) - (a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1)\cdot(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)}{(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)\cdot(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1) - (d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1)\cdot(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)}$$

At this point we should get rid of all the brackets and cancel out as much as possible.