

Extended Steinhart Heart's polynom (1) is just a polynom.

$$\frac{1}{T} = A + B \ln(R) + C \cdot (\ln(R))^2 + D \cdot (\ln(R))^3 \quad (1)$$

We can calculate it's coefficients A , B , C and D very elegantly. Because there are four coefficients, we will need to create a system of four equations like this:

$$\begin{aligned} \frac{1}{T_1} &= A + B \ln(R_1) + C \cdot (\ln(R_1))^2 + D \cdot (\ln(R_1))^3 \\ \frac{1}{T_2} &= A + B \ln(R_2) + C \cdot (\ln(R_2))^2 + D \cdot (\ln(R_2))^3 \\ \frac{1}{T_3} &= A + B \ln(R_3) + C \cdot (\ln(R_3))^2 + D \cdot (\ln(R_3))^3 \\ \frac{1}{T_4} &= A + B \ln(R_4) + C \cdot (\ln(R_4))^2 + D \cdot (\ln(R_4))^3 \end{aligned}$$

Now we can look inside any documentation for a well known NTC thermistor e.g. ./documentation-ntc.pdf where we find a table consisting of resistance measurments¹ at known temperatures. From that table we take four values which should be as far appart as possible.

$$R_1 = 329518 \, \Omega$$

$$R_2 = 19488 \, \Omega$$

$$R_3 = 2511 \, \Omega$$

$$R_4 = 503 \, \Omega$$

$$T_1 = -50 \, ^\circ\text{C} = -50 + 273.15K = 223.15K$$

$$T_2 = 8 \, ^\circ\text{C} = 8 + 273.15K = 281.15K$$

$$T_3 = 66 \, ^\circ\text{C} = 66 + 273.15K = 339.15K$$

$$T_4 = 125 \, ^\circ\text{C} = 125 + 273.15K = 398.15K$$

Now we know the values for resistance and temperature and can calculate the coefficients A , B , C and D . Before solving the system we first make it easier on ourselves and execute some substitutions².

¹We look at nominal values while ignoring minimum and maximum values.

²Right before the end we will revert these substitutions and calculate coefficients.

$\frac{1}{T_1} = a_1$	$\ln(R_1) = b_1$	$(\ln(R_1))^2 = c_1$	$(\ln(R_1))^3 = d_1$
$\frac{1}{T_2} = a_2$	$\ln(R_2) = b_2$	$(\ln(R_2))^2 = c_2$	$(\ln(R_2))^3 = d_2$
$\frac{1}{T_3} = a_3$	$\ln(R_3) = b_3$	$(\ln(R_3))^2 = c_3$	$(\ln(R_3))^3 = d_3$
$\frac{1}{T_4} = a_4$	$\ln(R_4) = b_4$	$(\ln(R_4))^2 = c_4$	$(\ln(R_4))^3 = d_4$

After the substitutions the system of equations becomes easier to read.

$$\begin{aligned}
a_1 &= A + Bb_1 + Cc_1 + Dd_1 \\
a_2 &= A + Bb_2 + Cc_2 + Dd_2 \\
a_3 &= A + Bb_3 + Cc_3 + Dd_3 \\
a_4 &= A + Bb_4 + Cc_4 + Dd_4
\end{aligned} \tag{2}$$

Now we start solving the system for A . First we express A from (2).

$$\begin{aligned}
A &= a_1 - Bb_1 - Cc_1 - Dd_1 \\
a_2 &= A + Bb_2 + Cc_2 + Dd_2 \\
a_3 &= A + Bb_3 + Cc_3 + Dd_3 \\
a_4 &= A + Bb_4 + Cc_4 + Dd_4
\end{aligned} \tag{3}$$

Then we continue by inserting (3) in all the rest.

$$\begin{aligned}
a_2 &= a_1 - Bb_1 - Cc_1 - Dd_1 + Bb_2 + Cc_2 + Dd_2 \\
a_3 &= a_1 - Bb_1 - Cc_1 - Dd_1 + Bb_3 + Cc_3 + Dd_3 \\
a_4 &= a_1 - Bb_1 - Cc_1 - Dd_1 + Bb_4 + Cc_4 + Dd_4
\end{aligned}$$

We use the brackets to shorten the system of equations and put all the parts without a coefficient to the left.

$$\begin{aligned}
a_2 - a_1 &= B(b_2 - b_1) + C(c_2 - c_1) + D(d_2 - d_1) \\
a_3 - a_1 &= B(b_3 - b_1) + C(c_3 - c_1) + D(d_3 - d_1) \\
a_4 - a_1 &= B(b_4 - b_1) + C(c_4 - c_1) + D(d_4 - d_1)
\end{aligned} \tag{4}$$

We now extract B from (4).

$$\begin{aligned}
B &= \frac{(a_2 - a_1) - C(c_2 - c_1) - D(d_2 - d_1)}{(b_2 - b_1)} \\
a_3 - a_1 &= B(b_3 - b_1) + C(c_3 - c_1) + D(d_3 - d_1) \\
a_4 - a_1 &= B(b_4 - b_1) + C(c_4 - c_1) + D(d_4 - d_1)
\end{aligned} \tag{5}$$

We insert (5) in all the rest.

$$a_3 - a_1 = \left[\frac{(a_2 - a_1) - C(c_2 - c_1) - D(d_2 - d_1)}{(b_2 - b_1)} \right] (b_3 - b_1) + C(c_3 - c_1) + D(d_3 - d_1) \tag{6}$$

$$a_4 - a_1 = \left[\frac{(a_2 - a_1) - C(c_2 - c_1) - D(d_2 - d_1)}{(b_2 - b_1)} \right] (b_4 - b_1) + C(c_4 - c_1) + D(d_4 - d_1) \tag{7}$$

Now we get rid of the fractions, so we multiply (6) and (7) by $(b_2 - b_1)$.

$$\begin{aligned}
(a_3 - a_1) \cdot (b_2 - b_1) &= [(a_2 - a_1) - C(c_2 - c_1) - D(d_2 - d_1)](b_3 - b_1) + C(c_3 - c_1) \cdot (b_2 - b_1) \\
&\quad + D(d_3 - d_1) \cdot (b_2 - b_1) \\
(a_4 - a_1) \cdot (b_2 - b_1) &= [(a_2 - a_1) - C(c_2 - c_1) - D(d_2 - d_1)](b_4 - b_1) + C(c_4 - c_1) \cdot (b_2 - b_1) \\
&\quad + D(d_4 - d_1) \cdot (b_2 - b_1)
\end{aligned}$$

We get rid of the square brackets.

$$\begin{aligned}
(a_3 - a_1) \cdot (b_2 - b_1) &= (a_2 - a_1) \cdot (b_3 - b_1) - C(c_2 - c_1) \cdot (b_3 - b_1) - D(d_2 - d_1) \cdot (b_3 - b_1) \\
&\quad + C(c_3 - c_1) \cdot (b_2 - b_1) + D(d_3 - d_1) \cdot (b_2 - b_1) \\
(a_4 - a_1) \cdot (b_2 - b_1) &= (a_2 - a_1) \cdot (b_4 - b_1) - C(c_2 - c_1) \cdot (b_4 - b_1) - D(d_2 - d_1) \cdot (b_4 - b_1) \\
&\quad + C(c_4 - c_1) \cdot (b_2 - b_1) + D(d_4 - d_1) \cdot (b_2 - b_1)
\end{aligned}$$

Now we remove the rest of the brackets by multiplying them.

$$\begin{aligned}
a_3b_2 - a_1b_2 - a_3b_1 + a_1b_1 &= a_2b_3 - a_1b_3 - a_2b_1 + a_1b_1 \\
&\quad - Cc_2b_3 + Cc_1b_3 + Cc_2b_1 - Cc_1b_1 - Dd_2b_3 + Dd_1b_3 + Dd_2b_1 - Dd_1b_1 \\
&\quad + Cc_3b_2 - Cc_1b_2 - Cc_3b_1 + Cc_1b_1 + Dd_3b_2 - Dd_1b_2 - Dd_3b_1 + Dd_1b_1 \\
a_4b_2 - a_4b_1 - a_1b_2 + a_1b_1 &= a_2b_4 - a_1b_4 - a_2b_1 + a_1b_1 \\
&\quad - Cc_2b_4 + Cc_1b_4 + Cc_2b_1 - Cc_1b_1 - Dd_2b_4 + Dd_1b_4 + Dd_2b_1 - Dd_1b_1 \\
&\quad + Cc_4b_2 - Cc_1b_2 - Cc_4b_1 + Cc_1b_1 + Dd_4b_2 - Dd_1b_2 - Dd_4b_1 + Dd_1b_1
\end{aligned}$$

We cancel as much as we can.

$$\begin{aligned}
a_3b_2 - a_1b_2 - a_3b_1 &= a_2b_3 - a_1b_3 - a_2b_1 \\
&\quad - Cc_2b_3 + Cc_1b_3 + Cc_2b_1 - Dd_2b_3 + Dd_1b_3 + Dd_2b_1 \\
&\quad + Cc_3b_2 - Cc_1b_2 - Cc_3b_1 + Dd_3b_2 - Dd_1b_2 - Dd_3b_1
\end{aligned} \tag{8}$$

$$\begin{aligned}
a_4b_2 - a_4b_1 - a_1b_2 &= a_2b_4 - a_1b_4 - a_2b_1 \\
&\quad - Cc_2b_4 + Cc_1b_4 + Cc_2b_1 - Dd_2b_4 + Dd_1b_4 + Dd_2b_1 \\
&\quad + Cc_4b_2 - Cc_1b_2 - Cc_4b_1 + Dd_4b_2 - Dd_1b_2 - Dd_4b_1
\end{aligned} \tag{9}$$

In (8) and (9) we move all the parts containing C to the left. We move all the rest to the right.

$$\begin{aligned}
Cc_2b_3 - Cc_1b_3 - Cc_2b_1 - Cc_3b_2 + Cc_1b_2 + Cc_3b_1 &= a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1 \\
&\quad - Dd_2b_3 + Dd_1b_3 + Dd_2b_1 \\
&\quad + Dd_3b_2 - Dd_1b_2 - Dd_3b_1
\end{aligned} \tag{10}$$

$$\begin{aligned}
Cc_2b_4 - Cc_1b_4 - Cc_2b_1 - Cc_4b_2 + Cc_1b_2 + Cc_4b_1 &= a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2 \\
&\quad - Dd_2b_4 + Dd_1b_4 + Dd_2b_1 \\
&\quad + Dd_4b_2 - Dd_1b_2 - Dd_4b_1
\end{aligned} \tag{11}$$

In (10) and (11) we express C on the left side and D on the right side.

$$\begin{aligned}
C(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1) &= a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1 \\
&\quad + D(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)
\end{aligned} \tag{12}$$

$$\begin{aligned}
C(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1) &= a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2 \\
&\quad + D(d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1)
\end{aligned} \tag{13}$$

We extract C from (12) and (13).

$$C = \frac{(a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1)}{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)} + D \frac{(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)}{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)} \quad (14)$$

$$C = \frac{(a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2)}{(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)} + D \frac{(d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1)}{(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)} \quad (15)$$

We take C from (14) and insert it in (15) to get a single equation.

$$\begin{aligned} & \frac{(a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1)}{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)} + D \frac{(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)}{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)} = \\ & \frac{(a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2)}{(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)} + D \frac{(d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1)}{(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)} \end{aligned} \quad (16)$$

We put all the elements containing D on the left and use brackets to only get one D .

$$D \left[\frac{(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)}{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)} - \frac{(d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1)}{(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)} \right] =$$

$$\frac{(a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2)}{(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)} - \frac{(a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1)}{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)}$$

Now we extract D .

$$D = \frac{\frac{(a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2)}{(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)}}{\frac{(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)}{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)}} - \frac{\frac{(a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1)}{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)}}{\frac{(d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1)}{(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)}}$$

We see some common large parts in the denominators of the nested fractions that we could cancel out. This is why we set the common denominator on the nested fractions.

$$D = \frac{\frac{(a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2) \cdot (c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1) - (a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1) \cdot (c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)}{(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1) \cdot (c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)}}{\frac{(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1) \cdot (c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1) - (d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1) \cdot (c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)}{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1) \cdot (c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)}}$$

Now the common denominators of the both nested fractions are the same! and they cancel each other out!

$$D = \frac{(a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2) \cdot (c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1) - (a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1) \cdot (c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)}{(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1) \cdot (c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1) - (d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1) \cdot (c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)}$$

At this point we should get rid of all the brackets and cancel out as much as possible, but we do not want to complicate too much as D is already extracted! This is why we just insert back the used substitutions with inserted resistance and temperature value pairs. But because this calculation can't fit on the paper we do it in the calculator³.

³We ommit the units.

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SpeedCrunch
Session Edit View Settings Help

a_1 = 1/223.15
= 0.0044812906116961685

a_2 = 1/281.15
= 0.00355682020273875156

a_3 = 1/339.15
= 0.00294854784018870706

a_4 = 1/398.15
= 0.00251161622504081376

b_1 = ln(329518)
= 12.70538625965721454144

b_2 = ln(19488)
= 9.87755417050962357077

b_3 = ln(2511)
= 7.82843635915758501151

b_4 = ln(503)
= 6.22059017009973920642

c_1 = (ln(329518))^2
= 161.42684000708634428961

c_2 = (ln(19488))^2
= 97.56607639135205775344

c_3 = (ln(2511))^2
= 61.2844158293804653485

c_4 = (ln(503))^2
= 38.69574206434150235403

d_1 = (ln(329518))^3
= 2050.99035496591836799595

d_2 = (ln(19488))^3
= 963.71420475966004223715

d_3 = (ln(2511))^3
= 479.76114912845468075243

d_4 = (ln(503))^3
= 240.71035271015773967182

D = ((a_2 b_4 - a_1 b_4 - a_2 b_1 - a_4 b_2 + a_4 b_1 + a_1 b_2) * (c_2 b_3 - c_1 b_3 - c_2 b_1 - c_3 b_2 + c_1 b_2 + c_3 b_1) - (a_2 b_3 - a_1 b_3 - a_2 b_1 - a_3 b_2 + a_1 b_2 + a_3 b_1) * (c_2 b_4 - c_1 b_4 - c_2 b_1 - c_4 b_2 + c_1 b_2 + c_4 b_1)) * ((d_3 b_2 - d_1 b_2 - d_3 b_1 - d_2 b_3 + d_1 b_3 + d_2 b_1) * (c_2 b_4 - c_1 b_4 - c_2 b_1 - c_4 b_2 + c_1 b_2 + c_4 b_1) - (d_4 b_2 - d_1 b_2 - d_4 b_1 - d_2 b_4 + d_1 b_4 + d_2 b_1) * (c_2 b_3 - c_1 b_3 - c_2 b_1 - c_3 b_2 + c_1 b_2 + c_3 b_1))
= -0.99704106394712860967

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And we get a value $D = -0.99704106394712860967$.

Now we can return back to a point where we had two equations, namely (12) and (13). This time we solve system of these two equations differently. We extract D from both of them.

$$\begin{aligned} C(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1) &= a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1 \\ &\quad + D(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1) \\ C(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1) &= a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2 \\ &\quad + D(d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1) \end{aligned}$$

$$D = C \frac{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)}{(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)} - \frac{(a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1)}{(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)} \quad (17)$$

$$D = C \frac{(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)}{(d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1)} - \frac{(a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2)}{(d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1)} \quad (18)$$

Now we take D from (17) and insert it in (18) to get a single equation for C .

$$\begin{aligned} C \frac{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)}{(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)} - \frac{(a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1)}{(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)} = \\ C \frac{(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)}{(d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1)} - \frac{(a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2)}{(d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1)} \end{aligned}$$

We put all the elements containing C on the left and use brackets to only get one C .

$$\begin{aligned} C \left[\frac{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)}{(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)} - \frac{(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)}{(d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1)} \right] = \\ \frac{(a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1)}{(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)} - \frac{(a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2)}{(d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1)} \end{aligned}$$

Now we extract C .

$$C = \frac{\frac{(a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1)}{(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)} - \frac{(a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2)}{(d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1)}}{\frac{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1)}{(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)} - \frac{(c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1)}{(d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1)}}$$

We see some common large parts in the denominators of the nested fractions that we could cancel out. This is why we set the common denominator on the nested fractions.

$$C = \frac{\frac{(a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1) \cdot (d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1) - (a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2) \cdot (d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)}{(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1) \cdot (d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1)}}{\frac{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1) \cdot (d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1) - (c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1) \cdot (d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)}{(d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1) \cdot (d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1)}}$$

Now the common denominators of the both nested fractions are the same! and they cancel each other out!

$$C = \frac{(a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1) \cdot (d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1) - (a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2) \cdot (d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)}{(c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1) \cdot (d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1) - (c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1) \cdot (d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)}$$

At this point we should get rid of all the brackets and cancel out as much as possible, but we do not want to complicate too much as C is already extracted! This is why we just insert back the used substitutions with inserted resistance and temperature value pairs. But because this calculation can't fit on the paper we do it in the calculator⁴.

```

((a_2b_3 - a_1b_3 - a_2b_1 - a_3b_2 + a_1b_2 + a_3b_1) * (d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1) - (a_2b_4 - a_1b_4 - a_2b_1 - a_4b_2 + a_4b_1 + a_1b_2) * (d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1)) / ((c_2b_3 - c_1b_3 - c_2b_1 - c_3b_2 + c_1b_2 + c_3b_1) * (d_4b_2 - d_1b_2 - d_4b_1 - d_2b_4 + d_1b_4 + d_2b_1) - (c_2b_4 - c_1b_4 - c_2b_1 - c_4b_2 + c_1b_2 + c_4b_1) * (d_3b_2 - d_1b_2 - d_3b_1 - d_2b_3 + d_1b_3 + d_2b_1))
= 0.00000943197911512071

```

And we get a value of $C = 0.00000943197911512071$.

⁴We ommit the units again.

Now we can return back to where we had three equations. We don't need all three of them. We simply input C and D in (5) to get equation for B . Because this equation would be very long, we will not derive it. We will only insert the numerical values for C , D to calculate the coefficient B .

$$B = \frac{(a_2 - a_1) - C(c_2 - c_1) - D(d_2 - d_1)}{(b_2 - b_1)}$$

```
B = ((a_2 - a_1) - C*(c_2 - c_1) - D*(d_2 - d_1)) / (b_2 - b_1)
= 383.35348690019995163549
```

And we get $B = 383.35348690019995163549$.

At the end we return back to where we had four equations. Again, we don't need all four of them. We simply input C , D and B in (3) to get equation for A . Because this equation would be very long, we will not derive it. We will only insert the numerical values for B , C and D to calculate the coefficient A .

$$A = a_1 - Bb_1 - Cc_1 - Dd_1$$

```
A = a_1 - B*b_1 - C*c_1 - D*d_1
= -2825.72956067693610481697
```

And we get $A = -2825.72956067693610481697$.