Cantor Diagonal Argument-false

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abstract

This analysis shows Cantor's diagonal argument published in 1891 cannot form a new sequence that is not a member of a complete list. The proof is based on the pairing of complementary sequences forming a binary tree model.

1. the argument

Assume a complete list L of random infinite sequences. Each sequence S is a unique infinite pattern of symbols from the set {0, 1}. A sample of a random list begins as:

S_1 $\underline{1}00101...$
S_2 $0\underline{1}0011...$
S_3 $11\underline{0}011...$
S_4 $100\underline{0}00...$
S_5 $0001\underline{1}1...$
S_6 $11100\underline{1}...$

A sequence p is formed from the diagonal elements (underlined) by applying the rule, if 0 then 1 else 0, to each position from left to right. The diagonal d=110011... is transformed via the substitution rule to the horizontal p = 001100...

1.1 Cantor's conclusion

Since p differs from each S in the sample by construction, it will differ from all S in the list L, therefore a new sequence p will be formed not in the list L. The set of integers N is not sufficient to count the list L. [1]

2. binary tree

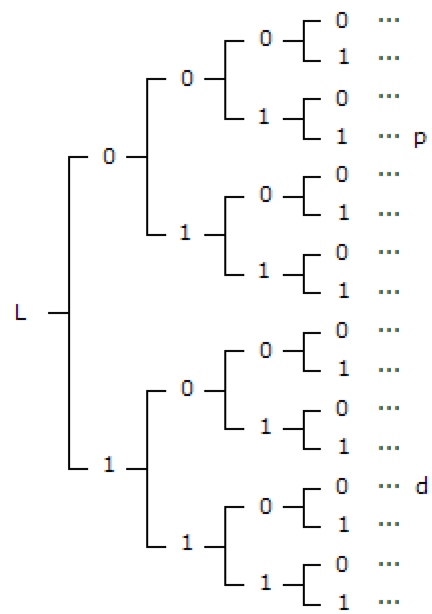

fig.1

The binary tree (fig.1) shows the beginning of all possible sequences, each corresponding to a unique linear path from left to right. All S must begin with 0 or 1, thus all would be contained in the tree if extended without limit. The tree therefore is a representation of L as defined in sec.1. Sequence p and its complement d are included in L as noted in fig.1. The tree is symmetrical relative to a horizontal line through L. If the tree is rotated 180° on the line, the symbols 0 and 1 are interchanged showing the pair of sequences d and p are complementary and mirror images. The number of paths at each position k equals 2^k. Before any symbol substitutions, d can exist anywhere in the list as S_r (fig.2).

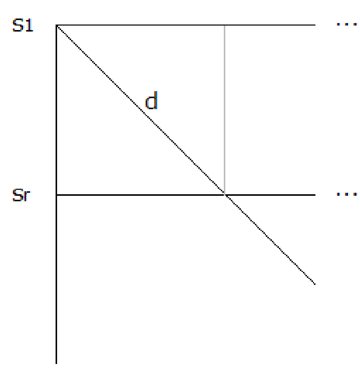

fig. 2

As d is transformed to p, the symbol at the intersection of S_r and d will differ as will all other positions. Thus d will be transformed to the complement of S_r.

the error

Cantor declares p an additional sequence to L because it differs from all S.

S_1	$\underline{1}00101...$	3
S_2	$0\underline{1}0011...$	1
S_3	$11\underline{0}011...$	0
S_4	$100\underline{0}00...$	3
S_5	$0001\underline{1}1...$	3
S_6	$11100\underline{1}...$	2

A random selection S_3 is compared to each S in the sample
for qty of differences (column 3).
The sequence S_3 differs from all S in the sample except one, itself. Since the sequences are unique, S_3 differs from all S in L except one, itself. Since S_3 is random, each S will differ from the remaining sequences, but all are existing elements of L.

conclusion

1. Each unique sequence S must differ from all other S in the list by at least one position, the greatest difference being all positions for S and its complement S', d and p in the example.
2. The diagonal d=110011... is already in the list as line 3, sec.1. Since his method only makes one comparison per sequence, it does not provide a means of detecting the complementary sequence, or that d is a duplication.
3. If d is not new, then neither is p, since they occur in complementary pairs.
4. If the unlimited sequences S_1, S_2, S_3, S_4,..., could be produced, a corresponding integer could be assigned from N, with the assurance that N is inexhaustible.

references

1. Cantor's Diagonal Argument, Wikipedia, Mar 2015