Nice proof here.The rigurous proof,actually.

For a plane movement

$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j}$
$\vec{v}(t) = x'(t)\vec{i} + y'(t)\vec{j}$
$\vec{a}(t) = x''(t)\vec{i} + y''(t)\vec{j}$

With the modulus

$|\vec{r}(t)| = +\sqrt{x^2(t) + y^2(t)}$
$|\vec{v}(t)| = +\sqrt{(x'(t))^2 + (y'(t))^2}$
$|\vec{a}(t)| = +\sqrt{(x''(t))^2 + (y''(t))^2}$

Okay?

Now assume that u want to express the same vector (i.e.the acceleration) in polar plane coordinates ρ and φ,but in the same $\left(\vec{i},\vec{j}\right)$ basis.

U know that

$x(t) = \rho(t)\cos\varphi(t)$
$y(t) = \rho(t)\sin\varphi(t)$

U'll simply compute that

$x'(t) = (\rho(t)\cos\varphi(t))' = \rho'(t)\cos\varphi(t) - \rho(t)\varphi'(t)\sin\varphi(t)$
$y'(t) = (\rho(t)\sin\varphi(t))' = \rho'(t)\sin\varphi(t) + \rho(t)\varphi'(t)\cos\varphi(t)$

and

$x''(t) = (x'(t))' = \rho''(t)\cos\varphi(t) - 2\rho'(t)\varphi'(t)\sin\varphi(t) - \rho(t)\varphi''(t)\sin\varphi(t) - \rho(t)(\varphi'(t))^2\cos\varphi(t)$

$y''(t) = (y'(t))' = \rho''(t)\sin\varphi(t) + 2\rho'(t)\varphi'(t)\cos\varphi(t) + \rho(t)\varphi''(t)\cos\varphi(t) - \rho(t)(\varphi'(t))^2\sin\varphi(t)$

Assume a circular movement.That means $\rho(t) = const. = R$,where R is the radius' circle.Then

$x'(t) = -R\varphi'(t)\sin\varphi(t)$
$y'(t) = R\varphi'(t)\cos\varphi(t)$

$x''(t) = -R\varphi''(t)\sin\varphi(t) - R(\varphi'(t))^2\cos\varphi(t)$
$y''(t) = R\varphi''(t)\cos\varphi(t) - R(\varphi'(t))^2\sin\varphi(t)$

Assume there is no angular acceleration $\alpha(t) =: \varphi''(t) = 0$.That means that the angular velocity $\omega(t) =: \varphi'(t) = const. = \omega$.The angles varies linearly in time $\varphi(t) = \omega t + \phi$.

Then the velocity & the acceleration have the components on the 2 axis

$x'(t) = -R\omega\sin(\omega t + \phi)$
$y'(t) = R\omega\cos(\omega t + \phi)$
$x''(t) = -R\omega^2\cos(\omega t + \phi)$
$y''(t) = -R\omega^2\sin(\omega t + \phi)$

The velocity's modulus is

$|\vec{v}\,(t)| = R\omega$

While the acceleration's

$|\vec{a}\,(t)| = R\omega^2$

The way i pictured the problem,the motion is anticlockwise.Define the **centripetal acceleration in an anticlockwise uniform circular movement** as the vector which has the components

$\vec{a}_{cp}\,(t) =: -R\omega^2 \cos\left(\omega t + \phi\right)\vec{i} - R\omega^2 \sin\left(\omega t + \phi\right)\vec{j}$

The velocity vector has the structure

$\vec{v}\,(t) = -R\omega \sin\left(\omega t + \phi\right)\vec{i} + R\omega \cos\left(\omega t + \phi\right)\vec{j}$

The position vector has the structure

$\vec{r}\,(t) = R\cos\left(\omega t + \phi\right)\vec{i} + R\sin\left(\omega t + \phi\right)\vec{j}$

Using these 3 last relations,u can easily prove that

$\vec{r}\,(t) \uparrow\downarrow \vec{a}_{cp}\,(t)$ and $\vec{v}\,(t) \perp \vec{a}_{cp}\,(t)$,which are typical for a circular movement.Due to the antiparallelism,the centripetal acceleration always points toward the center of the circle,hence the name.

Using the relations for the modulus of the velocity vector & the centripetal acceleration vector,u can easily prove that

$|\vec{a}_{cp}| = \frac{|\vec{v}|^2}{R}$