

Introductory Quantum Mechanics (FY507/FY521)

January 31, 2016

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CHAPTER 0

Course introduction

Welcome to the introductory course on quantum mechanics. The full semester version of the course is divided into two parts, which occasionally might also be called FY521 and FY522.

First part (FY521): will be about learning wave mechanics. We will usually meet three times a week for two hours. In the beginning of each of the first two times there will be a lecture introducing the topic of the day, and then after that you will work in groups either with extending the introduced material or applying it in exercises. The third time will be in the computer room Whitelab, where you will work on solving quantum mechanical problems numerically using the program Mathematica.

After each week, you will have to hand in your answers for the exercises marked with a * via SDU Assignment. This is compulsory, since the grade at the oral exam on April 6 or 8 will depend on the answers you hand in. The point of this oral exam is mainly to check that you have understood what you did. One reason for this is that you have to hand in your answers in groups of two, since otherwise the lecturers will have to spend too much time on correcting them. The file to be uploaded via SDU Assignment should be a PDF file, for instance containing a scan of hand written answers. Remember to write your names on the first page. We encourage the use of English, but Danish is also OK. Note, however, that the language at the oral exam will be English.

You might discover that you can find some of the derivations needed to answer the exercises written out in the textbook for the course. This is not due to an oversight of the lecturers. The philosophy behind the teaching in FY521 is that you understand the topics better by going through the fundamental derivations yourself. It requires some time to do it. But this is well spent given how unintuitive quantum mechanics is initially.

Second part (FY522): will focus more on formalism and analytical techniques to study quantum systems. This part will finish with a written exam in June.

The textbook that we will mainly refer you to is: D. J. Griffiths, *Introduction to Quantum Mechanics*, 2nd (international) edition, Pearson.

Outline of FY521:

Week	Subject	Literature
1	Basic formulation of quantum mechanics	Ch. 1
2	Solving the Schrödinger equation (I)	Ch. 2.1-2.2, 2.6
3	Solving the Schrödinger equation (II)	Ch. 2.3-2.4
4	Scattering of quantum particles	Notes
5	Delta-functions and periodic potentials	2.5.2, Notes
6	Formalism and Schrödinger equation in 3D	Ch. 3.1-3.4, 3.6, 4.1
7	Central potentials and the Hydrogen atom	4.1-4.2
8	Exam	*-exercises

CHAPTER 1

Basic formulation of Quantum Mechanics

Deadline for exercises: This weeks *-exercises should be uploaded via SDU Assingment before Tuesday 9/2 at 2pm.

1.1 Historical introduction

Subject keywords: Historical roots, particle/wave-duality, Schrödinger equation.

Literature: Parts of Chapter 1 of Griffiths, mainly Section 1.1 to 1.3.

Exercises: Are below.

1.1.1 *de Broglie wavelengths

Estimate some typical de Broglie wavelength for

- (a) an electron;
- (b) an apple.
- (c) Is it possible to measure an interference pattern for these objects in a practical experiment?

1.1.2 *Planar waves and Schrödingers equation

To study planar wave solutions to Schrödingers equation in one dimension

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi \quad (1.1)$$

for a free particle with $V(x) = 0$ assume a form $\Psi(x, t) = \exp[i(kx - \omega t)]$ of the wave function.

- (a) Find out how k and ω need to be related for this form to satisfy the Schrödinger equation (i.e., find the dispersion relation).
- (b) Write $\Psi_s = \sin(kx - \omega t)$ and $\Psi_c = \cos(kx - \omega t)$ as a linear combination of wave functions of the previous form.

1.1.3 Probability density function

Check whether the following functions are probability density functions. If a function fails to be a probability density function, explain why.

- (a) $f(x) = 1$ on $[0, 1]$
- (b) $f(x) = x$ on $[0, 1]$
- (c) $f(x) = \frac{3}{2}(x^2 - 1)$ on $[0, 2]$
- (d) $f(x) = \frac{3}{2}(1 - x^2)$ on $[0, 1]$
- (e) $f(x) = 1/x$ on $[0, e]$
- (f) $f(x) = \exp(x)$ on $[0, \ln 2]$

Define appropriate probability density functions for the following examples and estimate the probabilities to the 4th digit

- (g) Assuming that workers' salaries in a company are uniformly distributed between 10,000 USD and 40,000 USD per year, find the probability that a randomly chosen worker earns an annual salary between 14,000 USD and 20,000 USD.
- (h) The half-life of Carbon-14 is 5,730 years. What is the probability that a randomly selected Carbon-14 atom will not yet have decayed in 4,000 years' time?

1.1.4 *Continuous probability

If $p(x) = Cx \exp(-x/\lambda)$ is a probability density function over the interval $0 < x < +\infty$, find

- (a) the normalization constant C ;
- (b) the mean of x ;
- (c) the standard deviation of x ;
- (d) the most probable value (where the probability density is maximum) of x .