

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$$\text{IC: } T(x, 0) = T_i$$

$$\text{BC}_1: T(x \rightarrow \infty, t) = T_i$$

$$\text{BC}_2: -k \left. \frac{\partial T}{\partial x} \right|_{x=0} = h[T_\infty - T(0, t)]$$

$$\text{Similarity of variables: } \eta = \frac{x}{(4\alpha t)^{1/2}}$$

$$\frac{\partial T}{\partial x} = \frac{\partial T}{\partial \eta} \times \frac{\partial \eta}{\partial x} = \frac{dT}{d\eta} \times \frac{1}{(4\alpha t)^{1/2}}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial \left[\frac{\partial T}{\partial x} \right]}{\partial \eta} \times \frac{\partial \eta}{\partial x} = \frac{\partial^2 T}{\partial \eta^2} \times \frac{1}{4\alpha t}$$

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial \eta} \times \frac{\partial \eta}{\partial t} = \frac{\partial T}{\partial \eta} \times \frac{-x}{2t(4\alpha t)^{1/2}}$$

$$\frac{1}{4\alpha t} \times \frac{\partial^2 T}{\partial \eta^2} = \frac{1}{\alpha} \times \frac{-\eta}{2t} \times \frac{\partial T}{\partial \eta}$$

$$\frac{\partial^2 T}{\partial \eta^2} = -2\eta \frac{\partial T}{\partial \eta}$$

$$\text{IC: } T(\eta \rightarrow \infty) = T_i$$

$$\text{BC}_1: T(\eta \rightarrow \infty) = T_i$$

$$\text{BC}_2: -k \left. \frac{dT}{d\eta} \times \frac{1}{(4\alpha t)^{1/2}} \right|_{\eta=0} = h[T_\infty - T(\eta=0)]$$