

By definition of a scalar field  $\phi$ , one has  $\phi'(t', x', y', z') = \phi(t, x, y, z)$ , where the primed and the unprimed quantities are measured in two different inertial frames. Given a scalar field  $\phi$ , the quantity  $\square\phi$  is also independent of the inertial frame, where

$$\begin{aligned} \square &= \frac{\partial^2}{\partial t^2} - \Delta \\ \text{with } \Delta &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} , \end{aligned} \quad (1)$$

such that

$$\square\phi(t, x, y, z) = \square'\phi'(t', x', y', z') . \quad (2)$$

On the other hand,  $\Delta\phi$  is not relativistically invariant, and one can make a Taylor expansion in  $v/c$  such that

$$\Delta'\phi'(t', x', y', z') = \Delta\phi(t, x, y, z) + \frac{v^2}{c^2}F(t, x, y, z) + \dots , \quad (3)$$

where dots represent higher order terms in  $v/c$ . Therefore, when  $v \ll c$ , we have  $\Delta'\phi' \simeq \Delta\phi$  and the non-relativistic approximation is good.

**Question:** Consider the Lorentz transformation

$$\begin{aligned} ct' &= \gamma(ct - vx/c) \\ x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ \text{with } \gamma &= \frac{1}{\sqrt{1 - (v/c)^2}} , \end{aligned} \quad (4)$$

and calculate the function  $F$ .