

By definition of a scalar field ϕ , one has $\phi'(t', x', y', z') = \phi(t, x, y, z)$, where the primed and the unprimed quantities are measured in two different inertial frames. Given a scalar field ϕ , the quantity $\square\phi$ is also independent of the inertial frame, where

$$\begin{aligned}\square &= \frac{\partial^2}{\partial t^2} - \Delta \\ \text{with } \Delta &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} ,\end{aligned}\tag{1}$$

such that

$$\square\phi(t, x, y, z) = \square'\phi'(t', x', y', z') .\tag{2}$$

On the other hand, $\Delta\phi$ is not relativistically invariant, and one can make a Taylor expansion in v/c such that

$$\Delta'\phi'(t', x', y', z') = \Delta\phi(t, x, y, z) + \frac{v^2}{c^2}F(t, x, y, z) + \cdots ,\tag{3}$$

where dots represent higher order terms in v/c . Therefore, when $v \ll c$, we have $\Delta'\phi' \simeq \Delta\phi$ and the non-relativistic approximation is good.

Question: Consider the Lorentz transformation

$$\begin{aligned}ct' &= \gamma(ct - vx/c) \\ x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ \text{with } \gamma &= \frac{1}{\sqrt{1 - (v/c)^2}} ,\end{aligned}\tag{4}$$

and calculate the function F .