UNIVERSITY OF LONDON

BSc/MSci EXAMINATION May 2004

for Internal Students of Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant Examination for the Associateship

PHYSICS I Comprehensive Paper

For Third - and Fourth -Year Physics Students

Friday 21st May 2004: 10.00 to 13.00

Answer FIVE questions.

All questions carry equal marks.

Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Write your CANDIDATE NUMBER clearly on each of the FIVE answer books provided.

If an electronic calculator is used, write its serial number in the box at the top right hand corner of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the horizontal box on the front cover of its corresponding answer book.

Hand in FIVE answer books even if they have not all been used.

You are reminded that the Examiners attach great importance to legibility, accuracy and clarity of expression.

Turn over for questions

1. Consider the impact of a ball filled with gas at excess pressure p with a hard flat surface. Assume that the outer skin of the ball is flexible and that the pressure remains constant.

(i) If the outer skin of the ball is at some instant pushed in by contact with the surface by an amount x, find an approximate expression, in terms of x, for the area of contact, A, between the ball and the surface. Assume that x is much less than the radius of the ball, R.

[6 marks]

(ii) Hence find the restoring force on the ball and show that while the ball remains in contact with the surface its motion is described approximately by simple harmonic motion (SHM) at angular frequency $(2\pi Rp/M)^{1/2}$, where M is the mass of the ball.

[4 marks]

(iii) Using the above results, find an expression for the contact time when a ball bounces off a hard surface.

[2 marks]

(iv) The excess pressure in a football is roughly $p = 8.5 \times 10^4$ Pa. By making reasonable estimates for the values of the other relevant physical quantities, *estimate roughly* the contact time when a football bounces off a hard surface.

[3 marks]

(v) If the ball falls to the ground from a height of 5 m, estimate the value of x and comment on the validity of the SHM approximation in this case.

[5 marks]

[TOTAL 20 marks]

2. (i) A free-wheeling railway truck, mass M_0, is travelling without friction at speed V_0 along a straight horizontal track. Rain starts to fall at a rate R ($\mathrm{kg\,m^{-2}\,s^{-1}}$) and water accumulates in the truck through its open top which has area A. Show that the speed of the truck with its load of water varies with time t, after the start of the rain, according to:

$$V(t) = V_0 \left(1 + \frac{AR}{M_0} t\right)^{-1}$$

[4 marks]

[You may assume that the rain falls vertically and ignore any effect of rain hitting the front of the truck]

(ii) Find an expression for the kinetic energy of the truck as a function of time and comment on energy conservation.

[3 marks]

(iii) At time t_1 it stops raining and a drain plug is opened in the bottom of the truck (which is still moving) so that the water flows out until time t_2 when the truck is empty again. If the outflow rate is given by:

$$L(t) = L_0 \left(1 - \frac{t}{t_2}\right) \left(\mathrm{kg\,s^{-1}}\right)$$

find an expression for the mass of the truck in the period $t_1 < t < t_2$ and show that

$$\frac{\left({t_1}^2 + {t_2}^2\right)}{2t_1 t_2} = 1 + \frac{AR}{L_0}$$

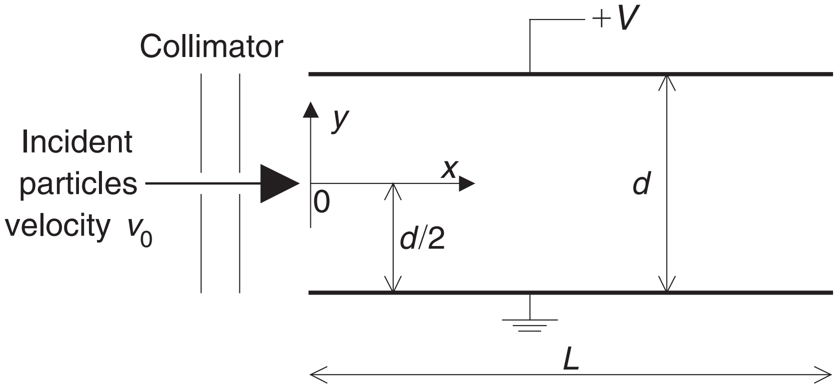
[6 marks]

(iv) Consider carefully the speed of the truck during $t_1 < t < t_2$ and sketch graphs to show how its mass, speed, momentum and kinetic energy vary with time from before $t = 0$ until after $t = t_2$.

[7 marks]

[TOTAL 20 marks]

3. A parallel plate capacitor can be used as an electrostatic analyser for measuring fluxes of energetic charged particles. A simplified version is shown in cross section in the Figure below. Energetic electrons and positively charged ions enter the capacitor through a collimator at $x = 0$, $y = 0$ with a velocity v_0 directed along the x-axis. The electrons' mass is m_e and their charge is $-$e, the mass of the ions is M_i, and their charge is $Q_\mathrm{i} = n\mathrm{e}$ where $n(> 0)$ is an integer. W_e and W_i denote their kinetic energies. The separation of the plates is d, their length is L, and their width is adequate to avoid fringe fields. The upper plate is at a potential $V(> 0)$ and the lower plate is grounded.



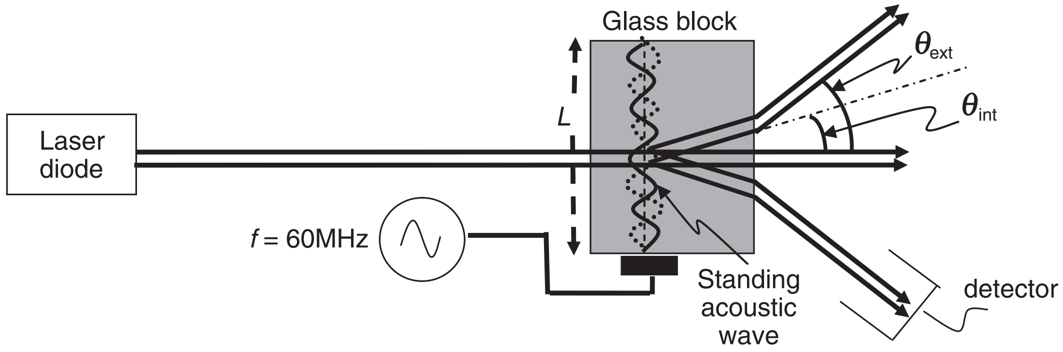
(i) State and justify the equations of motion of the electrons and ions in the analyser. In which direction will the electrons and positively charged ions be deflected?
[5 marks]

(ii) Calculate the time T taken for the electrons and ions to reach $x = L$ and calculate the position of the electrons and ions along the y-axis for $x = L$. Show that the ratio of the kinetic energy of the particles to their charge can be expressed as a function of the displacement D, measured from the x-axis at $x = L$, and taken to be a positive number for both electrons and ions. It can be assumed that $L \gg d$.
[8 marks]

(iii) A particle detector is placed at $x = L$ so that it can detect positive ions that have a displacement D between $d/4$ and $d/2$. On a sketch, show the position of the detector. Calculate the range of kinetic energy to charge ratio of the particles incident on this detector if d = 4 cm, $L = 10\,\mathrm{cm}$ and $V = 400\,\mathrm{V}$. Could this detector be used to detect protons with energy 1.5 keV? Could it detect alpha particles (fully ionised helium atoms) that travel at the same speed as the protons?
[7 marks]

[TOTAL 20 marks]

4. The device below uses an acoustic wave, which is created inside a block of glass, to deflect a laser beam. When the frequency, f, of the acoustic wave matches the standing wave resonance frequency of the glass block, the high wave amplitude results in alternating regions of strong compression and extension in the glass. These regions behave as a sinusoidal grating with a period equal to Λ, the acoustic wavelength, which diffracts the laser beam.



(i) Given that the laser diode is made from a semiconductor with a bandgap of 1.61 eV, calculate the optical wavelength of the light

(a) in air

(b) in the glass [4 marks]

(ii) A glass block of length $L = 5 \times 10^{-2}$ m is available. What is the minimum thickness of glass which must be removed from one of its ends to make a device which will work at a resonant frequency $f = 60$ MHz? [4 marks]

(iii) Calculate the angle, θ_{int}, through which the laser light is diffracted by the grating inside the glass. [2 marks]

(iv) Calculate θ_{ext}, the total deflection angle of the beam after it has left the device. [2 marks]

(v) Remembering that the amplitude of the diffraction grating oscillates in time, sketch a graph of intensity vs. time recorded at the light detector in the diagram. Label the time axis of this graph fully. [2 marks]

(vi) Laboratory measurements show that the sound *amplitude* in the block decays exponentially, with a time constant of 40 msec, after the electrical drive is switched off. Calculate the quality factor, Q, of the acoustic resonance. [3 marks]

(vii) Using your value for Q, calculate the frequency width, Δf, of the acoustic resonance at 60 MHz. Hence, assuming that temperature variations affect only the length, L, of the glass block, use the data below to estimate the temperature range over which this highly resonant device could be expected to work satisfactorily. [3 marks]

[TOTAL 20 marks]

Data :–

Speed of sound in glass $v_{\text{sound}} = 3.184 \times 10^3$ m sec^{-1}; Glass refractive index $n_{\text{glass}} = 1.43$;
Air refractive index $n_{\text{air}} = 1$; glass thermal expansion coefficient $\alpha_{\text{glass}} = 8.6 \times 10^{-6}$ K^{-1}.

5. A highly-charged ion consists of a nucleus of charge Ze with N electrons, each of charge $-e$, bound to it, such that $Z \gg N$. For a hydrogen-like ion, $N = 1$ and the atomic energy levels follow the pattern observed in hydrogen.

(i) Given that the Rydberg constant is roughly equivalent to 13.6 eV, find the binding energy of the ground state of the hydrogen-like ion of uranium ($Z = 92$) and calculate the energy and wavelength of the $n = 2$ to $n = 1$ transition in this species.
[5 marks]

(ii) The Lamb shift is a small increase of the ground state energy of an atomic system, which can be described using Quantum Electrodynamics. Its value is given by the formula:

$$\Delta E = (\alpha/\pi)(Z\alpha)^4 m_e c^2 F(Z\alpha)$$

where $F(Z\alpha)$ has the value 1.92 for $Z = 92$. Find the value of the Lamb shift for hydrogen-like uranium in eV.
[3 marks]

(iii) The Lamb shift could be determined by measuring the shift of the $n = 2$ to $n = 1$ transition energy from its theoretical value excluding the QED correction. Calculate the fractional change in transition energy arising from the Lamb shift and the percentage accuracy with which this change could be determined if the transition energy could be measured to an accuracy of 10 eV.
[3 marks]

(iv) Alternatively, the Lamb shift could be determined by ultra-high precision mass measurement. Write down an exact expression for the relativistic mass of a hydrogen-like uranium ion, $M(^{238}\mathrm{U}^{91+})$, in terms of the mass of the bare nucleus $M(^{238}\mathrm{U}^{92+})$, the mass of the electron, m_e, and the electron binding energy E. Hence find an expression for the fractional change in mass due to the presence of the Lamb shift. Given that mass measurements in this region are expected to achieve a relative accuracy in the region of 10^{-10}, to what percentage accuracy could such a measurement be used to determine the Lamb shift?
[5 marks]

(v) Comment on the types of physical processes which might take place when such a highly-charged ion in a low-energy beam interacts with a gas of neutral atoms.
[4 marks]

[TOTAL 20 marks]

Data :–
The electron mass m_e is equivalent to 511 keV/c^2.
The atomic mass unit u is equivalent to 932 MeV/c^2.

6. Let L_x, L_y and L_z denote the cartesian components of the orbital angular momentum of a quantum-mechanical particle.

(i) What is the classical definition of angular momentum, $\mathbf{L}$, in terms of position, $\mathbf{r} = (r_x, r_y, r_z)$, and momentum, $\mathbf{p} = (p_x, p_y, p_z)$?

[2 marks]

(ii) Using the correspondance principle, write down the operators L_x, L_y and L_z in the Schrodinger representation.

[6 marks]

(iii) Evaluate the commutators $[L_x, L_x]$, $[L_x, L_y]$ and $[L^2, L_x]$ where L^2 is the square of the total angular momentum operator. Of all the commutators between the L_x, L_y, L_z and L^2, which are non-zero?

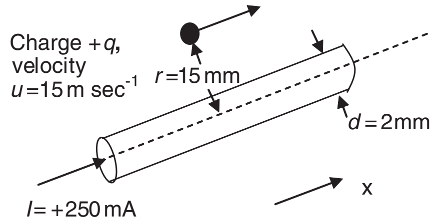
[6 marks]

(iv) In terms of the commutation relations from part (iii), explain why, notwithstanding that the angular momentum is a three-component vector, the eigenstates of angular momentum are characterized in terms of two quantities. What are the uncertainty relations $\Delta L_j \Delta L_k$ and $\Delta L_j \Delta L^2$?

[6 marks]

[TOTAL 20 marks]

7. A long, electrically neutral copper wire, of diameter $d = 2$ mm, carries a current I in the x-direction, which can be considered as being due to the motion of electrons, at uniform volume density, $n\,\mathrm{m}^{-3}$, moving with average velocity v, through an equal density of stationary positive charges.



(i) Derive an expression for the magnetic field distribution in the vicinity of the wire.
[3 marks]

(ii) A positive charge , $+q$, moves parallel to the wire, at a distance r from its axis, at a speed u, in the same direction as the current. Derive an expression for the force experienced by this charge and indicate its direction.
[2 marks]

(iii) Evaluate your answer to (ii) for $q = +0.3\,\mathrm{C}$, $r = 15\,\mathrm{mm}$, $U = 15\,\mathrm{m\,sec}^{-1}$, and $I = 250\,\mathrm{mA}$.
[2 marks]

(iv) Using the data below, calculate v, the velocity of the moving electrons in the copper wire.
[3 marks]

The whole system is now viewed in the rest frame of the charge q. In this frame the wire carries a net charge per unit length given by

$$\rho = \frac{ne\pi d^2}{4}\left[\gamma_p - \gamma_n\right]$$

where γ_p and γ_n are the Lorentz factors associated with the velocities of the positive charges and the negative electrons respectively.

(v) Briefly describe the physical origin of this charge on the wire.
[3 marks]

(vi) Using Gauss' law, derive an expression, in terms of ρ, for the electrostatic force experienced by the stationary charge Q.
[4 marks]

In the limit $u, v \ll \mathrm{c}$ and $v \ll u$, the difference in the Lorentz factors, $(\gamma_p - \gamma_n)$ can be shown to be equal to vu/c^2.

(vii) For the parameters above, evaluate the magnitude and direction of the electrostatic force experienced by Q, measured in its rest frame, *and comment on the result in relation to your answer to part (iii).*
[3 marks]

[TOTAL 20 marks]

Data:–
Density of free electrons in copper, $n = 8.47 \times 10^{28}\,\mathrm{m}^{-3}$.

8. The price X of an asset on the stock market fluctuates randomly from day to day. In a toy model, we assume that the price changes daily in one of two ways. The price X_i on day i may rise compared to the price X_{i-1} on the previous day $i-1$ by a multiplicative factor: $X_i/X_{i-1} = 1+\delta$ with probability p. Alternatively, it may fall by a factor $X_i/X_{i-1} = 1-\delta$ with probability $1-p$. The value of δ is constant. We also assume that the price changes on different days are independent random events.

An investor buys this asset at price X_0. She hopes to make a profit of 25%, that is, $(X - X_0)/X_0$ reaches 0.25.

(i) Assuming $\delta \ll 1$, show that the quantity $Y_i = \ln(X_i/X_0)$ changes daily by

$$\Delta_i \equiv Y_i - Y_{i-1} \simeq \quad +\delta \quad \text{or} \quad -\delta$$

with probabilities p and $1-p$ respectively. [1 mark]

(ii) What is the mean of Δ_i? What is its variance? [4 marks]

(iii) The price is X_N at N days after the purchase. What is the expected value for $Y_N = \ln(X_N/X_0)$? Show that the standard deviation of Y_N is $2\delta[p(1-p)N]^{1/2}$. (*Hint:* recall that means and variances can be added for independent events.) [4 marks]

(iv) Assume $p > 0.5$ so that the price has a rising trend. What is the expected number of days, N_{avg}, the investor has to wait for a profit of 25%? Evaluate your expression for N_{avg} for $p = 0.7$ and $\delta = 0.01$. [3 marks]

(v) After many days ($N \gg 1$), what is the probability distribution $p_N(Y)$ for the quantity Y_N? Sketch this distribution, showing clearly the position and width of any feature. State any results you have used from the theory of statistics. [4 marks]

(vi) Using the distribution in part (v), calculate how long the investor has to wait before she has a 84% chance of reaching her target of a 25% profit. Compare this with your answer to part (iv).
(You can assume that $N \gg 1$.) [4 marks]

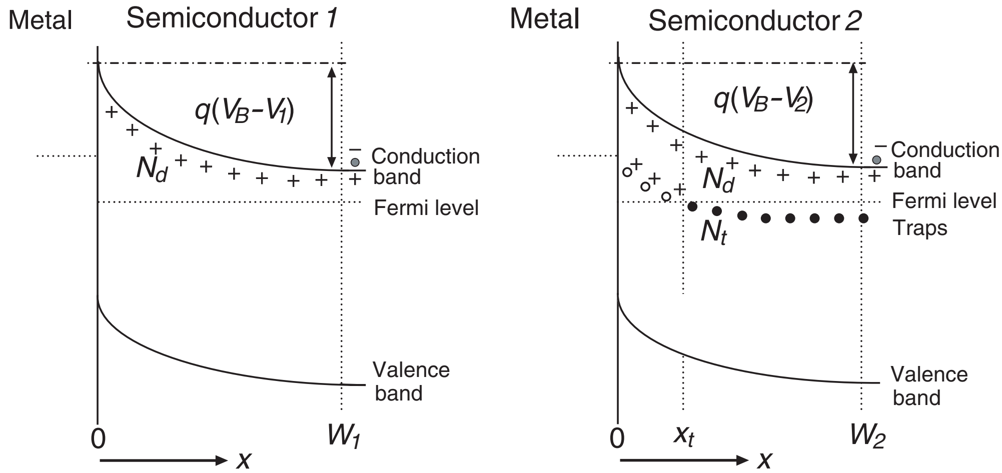
[TOTAL 20 marks]

You may find the following useful.
For $P(y)$ defined as

$$P(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-x^2/2} dx \ ,$$

$P(-2) = 0.02$, $P(-1) = 0.16$, $P(0) = 0.5$, $P(1) = 0.84$, $P(2) = 0.98$.

9. An n-type semiconductor 1 has a uniform donor dopant density N_d . All the dopant ions are fully ionised in the bulk of semiconductor 1, providing an electron density $n = N_d$ in the conduction band. When semiconductor 1 is placed in direct contact with a metal, due to the Fermi level difference between the materials a depletion region of width W_1 develops at the junction (see left hand Figure below). This is directly equivalent to the n side of a pn junction.



At zero applied voltage ($V_1 = 0$) the electrostatic potential difference between metal and semiconductor 1 is V_B. When a voltage is applied the potential difference between metal and semiconductor is $V_B - V_1$.

(i) In a simple model we assume that the depletion region ($0 < x < W_1$) contains a uniform positive charge density $\rho = +qN_d$ where q is the charge on the electron. Everywhere else in the bulk of the semiconductor 1 ($x > W_1$) the charge carrier density exactly balances the ionised dopants so that $\rho = 0$. Derive an expression for the electric field $E(x)$ in the depletion region of semiconductor 1 (take ε_s as the permittivity of semiconductor 1). Assume that $E(x = 0) = 0$. [5 marks]

(ii) Using the answer to (i), find the variation of the electric potential $V(x)$ with x in semiconductor 1. Assume that $V(x = 0) = V_B - V_1$ and $V(x = W_1) = 0$. Hence show that:

$$W_1 = \sqrt{\frac{2\varepsilon_s}{qN_d}(V_B - V_1)} \ .$$

[5 marks]

Semiconductor 2 is identical to semiconductor 1 (same N_d and ε_r) except that it contains a charge trap of uniform density N_t (see right hand Figure above). Semiconductor 2 also forms a junction with the same metal as semiconductor 1. At zero applied voltage ($V_2 = 0$) the trap is below the Fermi level and the electrostatic potential difference between metal and semiconductor 2 is also V_B. At a certain negative applied bias the trap is pulled above the Fermi level at the interface between $x = 0$ and $x = x_t$. Assume that the trap is donor type (neutral when full, positive when empty). Hence between $x = 0$ and $x = x_t$ the trap has positive uniform charge density $\rho_t = +qN_t$. For all $x > x_t$ the trap is neutral and $\rho_t = 0$.

(iii) For semiconductor *2* derive expressions for the electric field $E(x)$ in the depletion region between $x = 0$ and $x = x_t$ and between $x = x_t$ and $x = W_2$. Assume that $E(x = 0) = 0$.

[2 marks]

(iv) Assume that $V(x = 0) = V_B - V_2$ and $V(x = W_2) = 0$. By integrating across the whole depletion region, show that:

$$V_B - V_2 = \frac{q}{2\varepsilon_s}\left(N_d W_2^2 + N_t W_2 x_t\right).$$

[3 marks]

(v) The depletion region can be treated as a parallel plate capacitor of thickness W:

$$C = \varepsilon_s \big/ W$$

where the depletion capacitance per unit area C can be measured using a low-frequency capacitance meter. Derive an expression relating C to the applied bias (V_1 or V_2) for;

(a) Semiconductor *1* and

(b) Semiconductor *2* when the applied bias is large and negative so that $x_t \approx W$.

[2 marks]

(vi) The variation of C with applied bias is measured for both junctions. For semiconductor *1* it is found that:

$$1/C_1^2 = 1.198 \times 10^8 (V_B - V_1)$$

For semiconductor *2* at large, negative applied bias it is found that:

$$1/C_2^2 = 0.992 \times 10^8 (V_B - V_2)$$

where all values are in SI units. From these results, calculate values of N_d and N_t. What would the relationship between $1/C^2$ and V_2 in Semiconductor *2* be if the applied bias was small enough so that all the traps were below the Fermi level?

[$\varepsilon_s = 11.9 \times 8.854 \times 10^{-14}$ F/m.]

[3 marks]

[TOTAL 20 marks]

10. Write an essay about ONE of the following topics.

(i) Nuclear fusion is the only possible route to satisfying the world's energy requirements.

(ii) A manned mission to Mars is necessary to advance planetary science.

(iii) A-level physics should only be taught to those also studying A-level mathematics.

(iv) Experimental physics is dead; long live computer simulations.

(v) Physics experiments always work, just not always in the way that was intended.

(vi) Religious belief is incompatible with scientific determinism.

[TOTAL 20 marks]

End