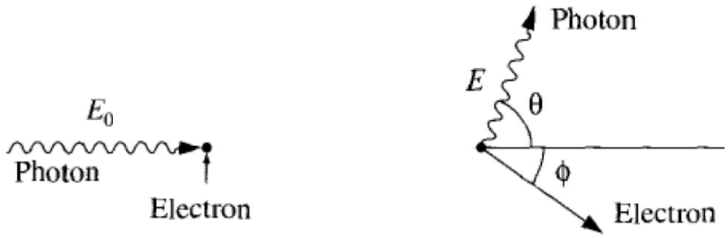
Derivation of the Compton effect



[I.6]

Lab frame. Conservation of four-momentum,

$$\left(E_0 + m_e, E_0, 0\right) = E\left(1, \cos\theta, \sin\theta\right) + \left(E_e, p_e \cos\phi, -p_e \sin\phi\right) \qquad [I.7]$$

Looking at [I.7], y-momentum balance says $E\sin\theta = p_e \sin\phi$. Use this in x-momentum balance, which says:

$$E_0 = E\cos\theta + p_e\cos\phi = E\cos\theta + p_e\sqrt{1-\sin^2\phi} = E\cos\theta + \sqrt{p_e^{\ 2} - (E\sin\theta)^2} \qquad [I.8]$$

Conservation of energy in [I.7] gives us information. Use $E_e \equiv \sqrt{m_e^{\ 2} + p_e^{\ 2}}$, and solve for $p_e^{\ 2}$ in [I.8] which, after using $\sin^2 + \cos^2 \equiv 1$, yields $p_e^{\ 2} = (E_0 - E\cos\theta)^2 + (E\sin\theta)^2 = E_0^{\ 2} + E^2 - 2E_0E\cos\theta$. Then, you get,

$$E_0 + m_e = E + E_e = E + \sqrt{m_e^{\ 2} + p_e^{\ 2}} = E + \sqrt{m_e^{\ 2} + E_0^{\ 2} + E^2 - 2E_0E\cos\theta} \quad \overset{\text{lots of algebra}}{\leftrightarrow} \quad E = \frac{1}{\frac{1-\cos\theta}{m} + \frac{1}{E_0}} \qquad [I.9]$$

The [I.9] immediately implies $\lambda' - \lambda = \frac{h}{mc}(1-\cos\theta)$ when we rescind $c = 1$. This formula is used above as if given.