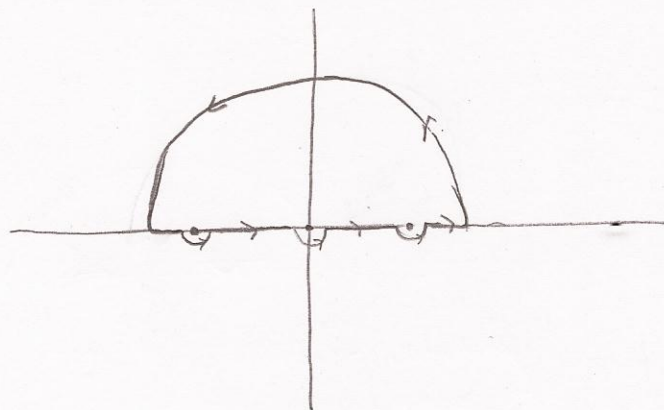


$$\int_0^{\infty} \frac{\sin(\pi x)}{x(1-x^2)} dx$$



change to complex

$$\text{Im} \left[ \frac{1}{2} \oint \frac{e^{i\pi z}}{z(1-z^2)} dz \right] \quad - \text{imaginary part because of sine}$$

simple pole @  $z=0$

simple poles @  $z = \pm 1$

Residue at  $z=0$

$$\lim_{z \rightarrow 0} \frac{\frac{1}{2} e^{i\pi z} \cdot z}{z(1-z^2)} = \frac{1}{2}$$

Residue at  $z=1$

$$\lim_{z \rightarrow 1} \frac{\frac{1}{2} e^{i\pi z} (\cancel{1-z})}{z(\cancel{1-z})(1+z)} = \frac{1}{4} e^{i\pi} = -\frac{1}{4}$$

Residue at  $z=-1$

$$\lim_{z \rightarrow -1} \frac{\frac{1}{2} e^{i\pi z} (\cancel{1+z})}{z(\cancel{1+z})(1-z)} = -\frac{1}{4} e^{-i\pi} = \frac{1}{4}$$

$$\text{Integral} = \text{Im} \left\{ \pi i \sum \text{residues enclosed} \right\}$$

$$\text{Im} \left\{ i\pi \left( \frac{1}{2} - \frac{1}{4} + \frac{1}{4} \right) \right\}$$

$$= \left( \frac{\pi}{2} \right) \quad ? \text{ answer should be } \pi \text{ according to Mathematica}$$