

$$|\Omega\rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} \left( e^{-iE_0(t_0-(-T))} \langle \Omega|0\rangle \right) U(t_0, -T)|0\rangle \quad (1)$$

Take the Hermitian Adjoint of both sides.

$$\langle \Omega| = \lim_{T \rightarrow \infty(1-i\epsilon)} \langle 0|U^\dagger(t_0, -T) \left( e^{iE_0(t_0-(-T))} \langle 0|\Omega\rangle \right)^{-1} \quad (2)$$

Equation 4.25 is

$$U(t, t') = e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \quad (3)$$

Therefore,  $U(a, b) = U^\dagger(b, a)$  (disregarding for a moment the issue of implicit time ordering in the definition of  $U$ ). So, eqn (2) becomes

$$\langle \Omega| = \lim_{T \rightarrow \infty(1-i\epsilon)} \langle 0|U(-T, t_0) \left( e^{iE_0(t_0-(-T))} \langle 0|\Omega\rangle \right)^{-1} \quad (4)$$

Also,  $U(-T, t_0) = U(-T, T)U(T, t_0)$  so,

$$\langle \Omega| = \lim_{T \rightarrow \infty(1-i\epsilon)} \langle 0|U(-T, T)U(T, t_0) \left( e^{iE_0(t_0-(-T))} \langle 0|\Omega\rangle \right)^{-1} \quad (5)$$

Substituting  $U(-T, T) = e^{iH_0(-T-t_0)} e^{2iHT} e^{-H_0(-T-t_0)}$ , eqn (5) becomes

$$\langle \Omega| = \lim_{T \rightarrow \infty(1-i\epsilon)} \langle 0|e^{iH_0(-T-t_0)} e^{iH(2T)} e^{-H_0(-T-t_0)} U(T, t_0) \left( e^{iE_0(t_0-(-T))} \langle 0|\Omega\rangle \right)^{-1} \quad (6)$$

Using  $H_0|0\rangle = 0$ , this can be rewritten as

$$\langle \Omega| = \lim_{T \rightarrow \infty(1-i\epsilon)} \langle 0|e^{iH(2T)} e^{-H_0(-T-t_0)} U(T, t_0) \left( e^{iE_0(t_0-(-T))} \langle 0|\Omega\rangle \right)^{-1} \quad (7)$$

**Doubtful step: If we assume that**  $\langle 0|e^{iH(2T)} = \langle 0|e^{iE_0(2T)}$ , then the  $e^{iE_0(2T)}$  factor goes through and we get

$$\begin{aligned} \langle \Omega| &= \lim_{T \rightarrow \infty(1-i\epsilon)} \langle 0|e^{-H_0(-T-t_0)} U(T, t_0) \left( e^{-iE_0(2T)} e^{iE_0(t_0-(-T))} \langle 0|\Omega\rangle \right)^{-1} \\ &= \lim_{T \rightarrow \infty(1-i\epsilon)} \langle 0|U(T, t_0) \left( e^{-iE_0(T-t_0)} \langle 0|\Omega\rangle \right)^{-1} \end{aligned}$$

as desired.

However, this is not strictly correct, even for large  $T$  because the starting point for this derivation was the assumption that there is some overlap of  $|\Omega\rangle$  with  $|0\rangle$ , that is,

$$e^{-iHT}|0\rangle = e^{-iE_0T}|\Omega\rangle\langle\Omega|0\rangle + \sum_{n \neq 0} e^{-iE_nT}|n\rangle\langle n|0\rangle \quad (8)$$

So, in deriving the expression above, we have not accounted for the overlap term  $\langle 0|\Omega\rangle$  which appears in the Hermitian Adjoint of eqn (8):

$$\langle 0|e^{iHT} = e^{iE_0T}\langle 0|\Omega\rangle\langle\Omega| + \sum_{n \neq 0} e^{+iE_nT}\langle 0|n\rangle\langle n| \quad (9)$$