This two-part problem is from O'Neill's Elementary Differential Geometry, section 2.5. It is in the second statement of the second part where I'm unclear, beginning with "Thus."

Full problem:

Let W be a vector field defined on a region containing a regular curve a. Then $W \circ a$ (i.e. W composed with a) is a vector field on a called the restriction of W to a.

1. Prove that $\nabla_{a'(t)} W = (W(a))'(t)$.

2. Deduce that the straight line in Definition 5.1 (below) may be replaced by any curve with initial velocity v. Thus the derivative Y' of a vector field Y on a curve a is (almost) $\nabla_{a'} Y$.

Definition 5.1. Let W be a vector field on $\mathbb{R}^3$, and let v be a tangent vector to $\mathbb{R}^3$ at the point p. The the covariant derivative of W with respect to v is the tangent vector $(W(p + tv))'(0)$ at the point p.

The following definition is useful for part (2) since it distinguishes between a vector field and a vector field on a curve.

Definition 2.2. A vector field on a curve a from I to $\mathbb{R}^3$ is a function that assigns to each number t in I a tangent vector $Y(t)$ to $\mathbb{R}^3$ at the point $a(t)$.

My attempt, and where I'm stuck:

Part (1) was fairly straight-forward, using the definitions of covariant derivative and what it means to differentiate a composition.

Part (2) has two parts. My approach to the first part is the following, and I believe it to be correct. The idea is to define a function on curves $a(t)$ and show that it agrees with the covariant derivative with respect to a vector v at a point p for all curves $a(t)$ such that $a(0) = p$ and $a'(0) = v$. Part (1) can be used to show the function is well-defined and that it indeed equals the covariant.

The second part, starting at "Thus" is where I'm having trouble. It's with the use of the word "almost." To me, if Y is a vector field on a curve a, then using the definition of covariant, straight-line or otherwise, makes no sense because $Y \circ a$ is not defined (i.e., Y is not defined on $\mathbb{R}^3$, only $\mathbb{R}$). So I thought this might be the almost part. However, what's 'almost' about it? I was thinking that perhaps given Y on a that a vector field $\overline{Y}$ could be defined such that $\overline{Y} \circ a = Y$, and that then Y' would equal the covariant of $\overline{Y}$ instead of Y. However, I don't think I can 'always' define such a $\overline{Y}$.