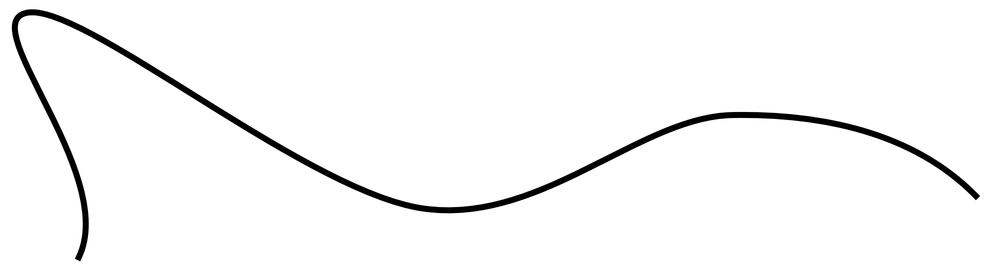
# Piecewise parametric cubic curve



- I created this curve with the **Insert > Shapes > Curve** function of PowerPoint.
- I cliked at five points and these points were interpolated to generate a curve.

# Parametric cubic curve

$$x(t) = a_x t^3 + b_x t^2 + c_x t + d_x$$
$$y(t) = a_y t^3 + b_y t^2 + c_y t + d_y$$

Or

$$\mathbf{S}(t) = \mathbf{a}t^3 + \mathbf{b}t^2 + \mathbf{c}t + \mathbf{d}$$

$$\mathbf{S}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, \text{ etc.}$$

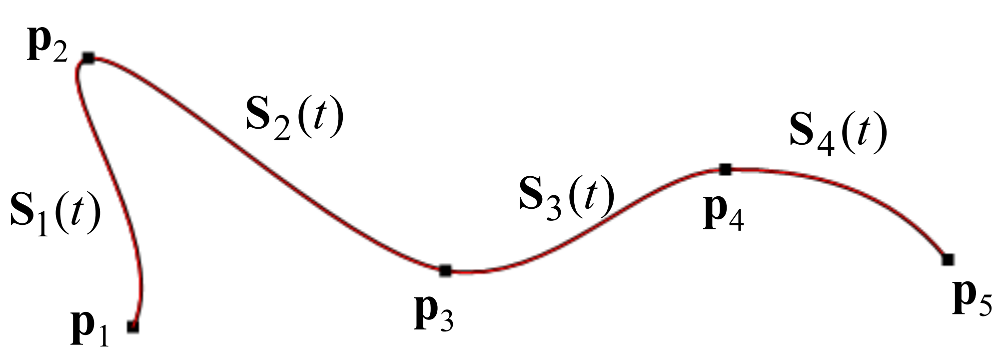
This curve is more complex than $y(x) = ax^3 + bx^2 + cx + d$

Imagine solving $x(t)$ for $t$ and plugging it into $y(t)$.

**We usually let *t* vary from 0 to 1.**

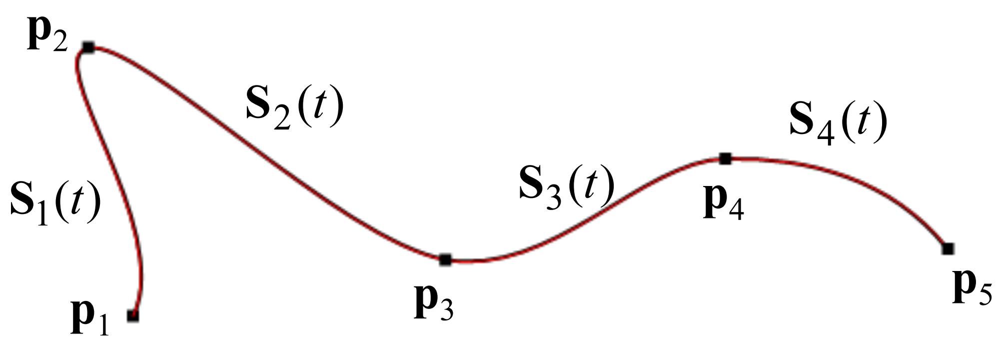
# Piecewise parametric cubic curve

Each curve between two points is a parametric cubic curve.




$$\mathbf{S}_1(0) = \mathbf{p}_1, \mathbf{S}_1(1) = \mathbf{p}_2$$
$$\mathbf{S}_2(0) = \mathbf{p}_2, \mathbf{S}_2(1) = \mathbf{p}_3$$
$$\ldots$$

Given the points, how can we determine the coefficients of the curves?




Each curve has to pass two points. → $2n$ vector equations

$$\mathbf{S}_1(0) = \mathbf{p}_1, \mathbf{S}_1(1) = \mathbf{p}_2$$
$$\mathbf{S}_2(0) = \mathbf{p}_2, \mathbf{S}_2(1) = \mathbf{p}_3$$
$$\dots$$

***n* is the number of curves.**
**The number of points is *n*+1.**

The first derivatives are continuous at each internal point. → $n$-1 vector equations

$$\mathbf{S}_1'(1) = \mathbf{S}_2'(0)$$
$$\mathbf{S}_2'(1) = \mathbf{S}_3'(0)$$
$$\dots$$

The second derivatives are continuous at internal each point. → $n$-1 vector equations

$$\mathbf{S}_1''(1) = \mathbf{S}_2''(0)$$
$$\mathbf{S}_2''(1) = \mathbf{S}_3''(0)$$
$$\ldots$$

We can specify conditions at the two end points. → 2 vector equations

Specifying the first derivative (tangent) $\quad \mathbf{S}_1'(0) = \mathbf{m}_s, \quad \mathbf{S}_n'(1) = \mathbf{m}_e$

Specifying the second derivative is zero. $\quad \mathbf{S}_1''(0) = 0, \quad \mathbf{S}_n''(1) = 0$

Specifying the second derivative $\quad \mathbf{S}_1''(0) = \mathbf{S}_1''(1), \quad \mathbf{S}_n''(1) = \mathbf{S}_n''(0)$

4$n$ vector equations in total for 4$n$ vector unknowns

Each curve has to pass two points. → $2n$ vector equations

$$\mathbf{S}_1(0) = \mathbf{p}_1 \quad d_{1x} = x_1$$
$$d_{1y} = y_1$$

$$\mathbf{S}_1(1) = \mathbf{p}_2 \quad a_{1x} + b_{1x} + c_{1x} + d_{1x} = x_2$$
$$a_{1y} + b_{1y} + c_{1y} + d_{1y} = y_2$$

$$\mathbf{S}_2(0) = \mathbf{p}_2 \quad d_{2x} = x_2$$
$$d_{2y} = y_2$$

$$\mathbf{S}_2(1) = \mathbf{p}_3 \quad a_{2x} + b_{2x} + c_{2x} + d_{2x} = x_3$$
$$a_{2y} + b_{2y} + c_{2y} + d_{2y} = y_3$$

$$x(t) = a_x t^3 + b_x t^2 + c_x t + d_x$$
$$y(t) = a_y t^3 + b_y t^2 + c_y t + d_y$$

The first derivatives are continuous at each internal point. → $n$-1 vector equations

$$\mathbf{S}'_1(1) = \mathbf{S}'_2(0) \quad 3a_{1x} + 2b_{1x} + c_{1x} = c_{2x}$$
$$3a_{1y} + 2b_{1y} + c_{1y} = c_{2y}$$

$$\mathbf{S}'_2(1) = \mathbf{S}'_3(0) \quad 3a_{2x} + 2b_{2x} + c_{2x} = c_{3x}$$
$$3a_{2y} + 2b_{2y} + c_{2y} = c_{3y}$$

$$x'(t) = 3a_x t^2 + 2b_x t + c_x$$
$$y'(t) = 3a_y t^2 + 2b_y t + c_y$$

The second derivatives are continuous at internal each point. $\rightarrow$ $n$-1 vector equations

$$x''(t) = 6a_x t + 2b_x$$
$$y''(t) = 6a_y + 2b_y$$

$$\mathbf{S}_1''(1) = \mathbf{S}_2''(0) \qquad 6a_{1x} + 2b_{1x} = b_{2x}$$
$$6a_{1y} + 2b_{1y} = b_{2y}$$

$$\mathbf{S}_2''(1) = \mathbf{S}_3''(0) \qquad 6a_{2x} + 2b_{2x} = b_{3x}$$
$$6a_{2y} + 2b_{2y} = b_{3y}$$

Specifying the second derivative is zero.

$$\mathbf{S}_1''(0) = 0 \qquad 2b_{1x} = 0$$
$$2b_{1y} = 0$$

$$\mathbf{S}_n''(1) = 0 \qquad 6a_{nx} + 2b_{nx} = 0$$
$$6a_{ny} + 2b_{ny} = 0$$

$$
\begin{bmatrix}
0 & 0 & 0 & 1 & & & & & & & & & & & & & \\
1 & 1 & 1 & 1 & & & & & & & & & & & & & \\
& & & & 0 & 0 & 0 & 1 & & & & & & & & & \\
& & & & 1 & 1 & 1 & 1 & & & & & & & & & \\
& & & & & & & & & \cdots & & & & & & & \\
3 & 2 & 1 & 0 & 0 & 0 & -1 & 0 & \cdots & & & & & & & & \\
& & & & 3 & 2 & 1 & 0 & 0 & 0 & -1 & 0 & & & & & \\
& & & & & & & & & \cdots & & & & & & & \\
6 & 2 & 0 & 0 & 0 & -2 & 0 & 0 & & & & & & & & & \\
& & & & 6 & 2 & 0 & 0 & 0 & -2 & & & & & & & \\
& & & & & & & & & \cdots & & & & & & & \\
0 & 2 & & & & & & & & & & & & & & & \\
& & & & & & & & & & & & & 6 & 2 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
a_{1x} \\ b_{1x} \\ c_{1x} \\ d_{1x} \\ a_{2x} \\ b_{2x} \\ c_{2x} \\ d_{2x} \\ \vdots \\ \\ \\ c_{nx} \\ d_{nx}
\end{bmatrix}
=
\begin{bmatrix}
x_1 \\ x_2 \\ x_2 \\ x_3 \\ \vdots \\ 0 \\ 0 \\ \\ 0 \\ 0 \\ \\ 0 \\ 0
\end{bmatrix}
$$

Blank spaces are mostly zeros.

By solving this with Gauss(), we get the coefficients of $x$ equations.

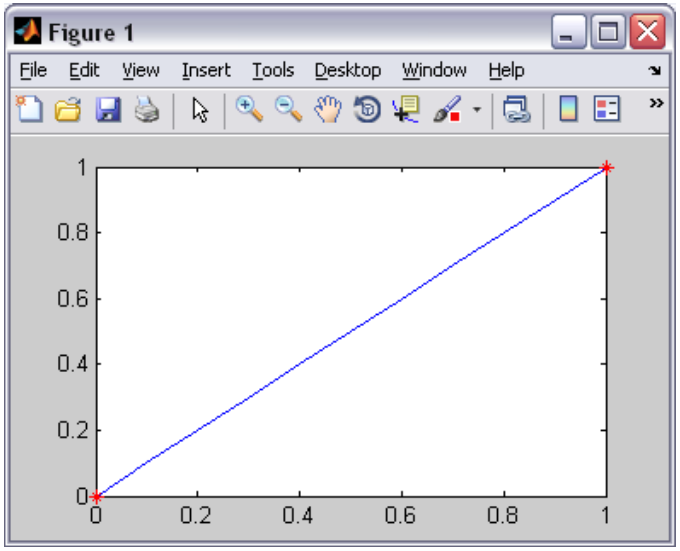
Then you can draw curves by calculating x and y for different t values.

For one piecewise cubic curve,

```
for i = 1:11
    t = 0.1*(i-1)
    xx(i) = ....
    yy(i) = .....
end
plot(xx,yy);
```

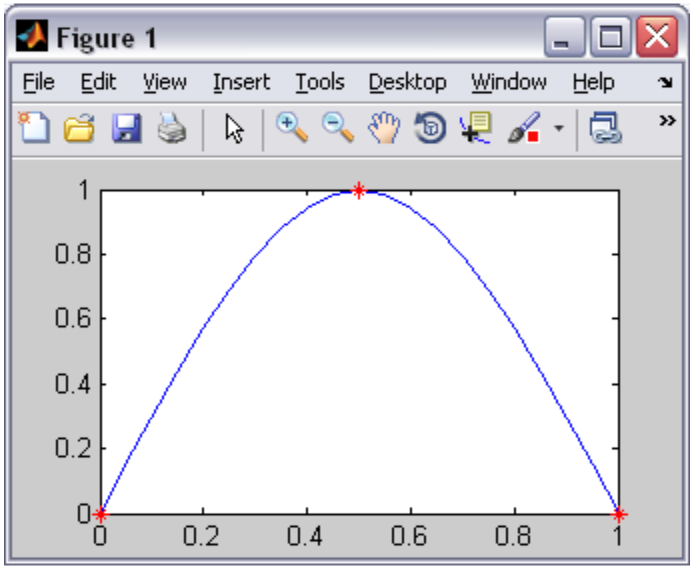
draw_line.m

```
clear x
clear y
x(1)=0;
y(1)=0;
x(2)=1;
y(2)=1;
cspline_curve(x,y);
```



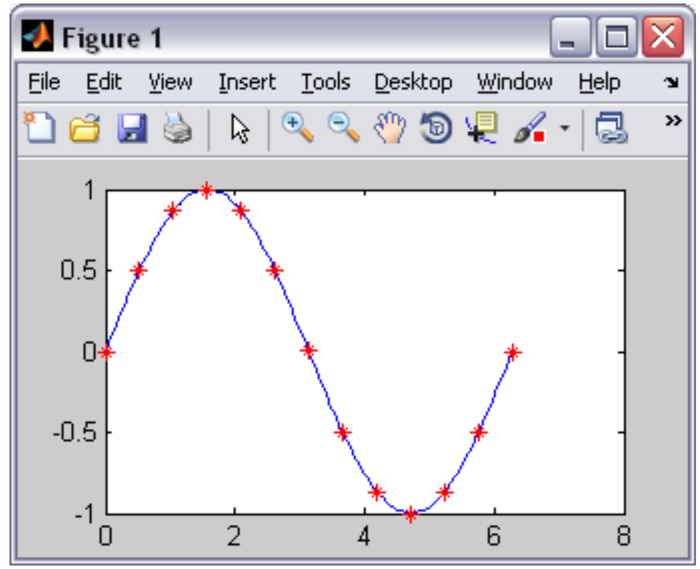

draw_tri.m

```
clear x
clear y
x(1)=0;
y(1)=0;
x(2)=0.5;
y(2)=1;
x(3)=1;
y(3)=0;
cspline_curve(x,y);
```



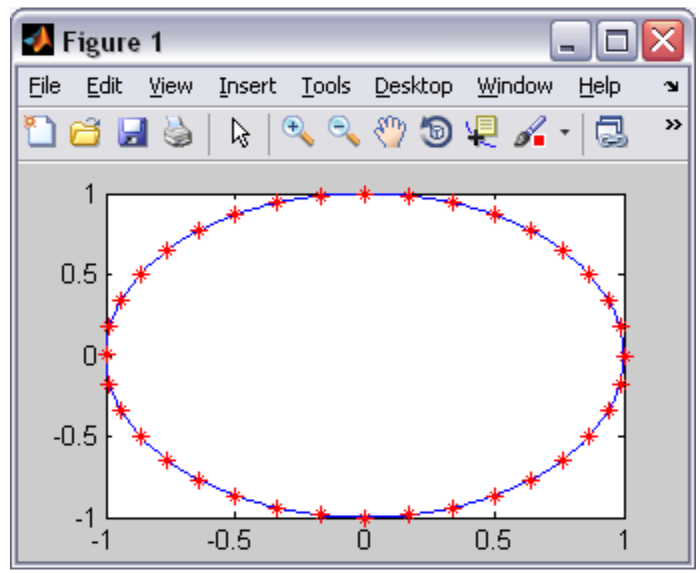
draw_sine.m

```
clear ang
clear y
i=0:12;
ang = 2*pi/12*i;
y = sin(ang);
cspline_curve(ang,y);
```



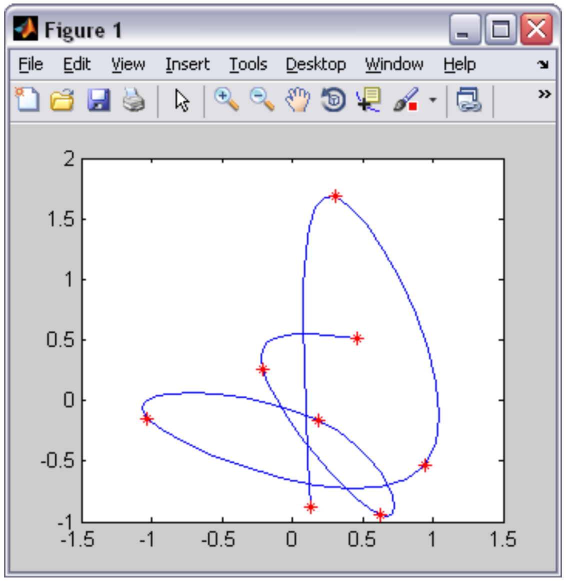

draw_circle.m

```
clear x
clear y
i=0:36;
ang=pi/180*i*10;
x=cos(ang);
y=sin(ang);
cspline_curve(x,y);
```



draw_random.m

```
clear x
clear y
x=random('Normal',0,1,1,8);
y=random('Normal',0,1,1,8);
cspline_curve(x,y);
```



```
clear x
clear y
i=0:36;
ang=pi/180*i*10;
x=cos(ang);
y=sin(ang);
cspline_curve(x,y);
```

Your task is to write this program.

draw_line.m, draw_tri.m, draw_sine.m, draw_circle.m, and draw_random.m are provided.