CURVATURE

We consider the Riemannian curvature tensor in spherical coordinates. That is we evaluate this rank four tensor in three dimensions.

$$R^{\alpha}{}_{\beta\gamma\delta} = \frac{d(\Gamma^{\alpha}{}_{\beta\delta})}{dx^{\gamma}} - \frac{d(\Gamma^{\alpha}{}_{\beta\gamma})}{dx^{\delta}} + \Gamma^{\alpha}{}_{\gamma\varepsilon}\Gamma^{\varepsilon}{}_{\beta\delta} - \Gamma^{\alpha}{}_{\delta\varepsilon}\Gamma^{\varepsilon}{}_{\beta\gamma}$$

Each index runs over three values.

For γ=δ

$$R^{\alpha}{}_{\beta\gamma\gamma} = 0$$

Again,

$$R_{\alpha\beta\gamma\gamma} = g_{\alpha\mu}R^{\mu}{}_{\beta\gamma\gamma} = 0$$

$$R_{\alpha\beta\gamma\gamma} = R_{\gamma\gamma\alpha\beta} = 0$$

The above relations are true for all three dimensional coordinates. But for spherical coordinates,

$$R_{\gamma\alpha\gamma\beta} = 0$$

$$R_{\gamma\alpha\beta\gamma} = 0$$

In view of the above it may be concluded that the Riemannian Christoffel curvature tensor has all its components zero if considered in the spherical coordinates.

Now if I have a sphere I find the Riemannian tensor zero at all points on it!