

# Initialisation

Clear all variables, just in case. This improves reusability as well

```
In[240]:= ClearAll["Global`*"];
```

Initialise the metric according to the form we found.

```
In[241]:= M = Table[0, {i, 4}, {j, 4}];
```

```
In[242]:= M[[1, 1]] = m11; M[[2, 2]] = m22; M[[3, 3]] = m33; M[[4, 4]] = m44; M[[2, 3]] = m23;
M[[2, 4]] = m24; M[[3, 2]] = m23; M[[3, 4]] = m34; M[[4, 2]] = m24; M[[4, 3]] = m34;
```

Create an abstract version of the inverse metric.

```
In[243]:= Minv = Table[0, {i, 4}, {j, 4}];
```

```
In[244]:= Minv[[1, 1]] = n11; Minv[[2, 2]] = n22; Minv[[3, 3]] = n33;
Minv[[4, 4]] = n44; Minv[[2, 3]] = n23; Minv[[2, 4]] = n24;
Minv[[3, 2]] = n23; Minv[[3, 4]] = n34; Minv[[4, 2]] = n24; Minv[[4, 3]] = n34;
```

Structure constants, the first index is an upper index, the others are lower indices.

```
In[245]:= F = Table[0, {i, 4}, {j, 4}, {k, 4}];
```

# Defining the structure constants

Grouped by type

```
In[246]:= F[[1, 2, 3]] = 0; F[[1, 2, 4]] = 0; F[[1, 3, 4]] = 1;
```

```
In[247]:= F[[2, 1, 2]] = 0; F[[3, 1, 3]] = 0; F[[4, 1, 4]] = 0;
```

```
In[248]:= F[[2, 1, 3]] = 1; F[[3, 1, 4]] = 1;
```

## Apply symmetries automatically

```
In[249]:= F[[1, 2, 3]] = -F[[1, 2, 3]];
F[[1, 4, 2]] = -F[[1, 2, 4]]; F[[1, 4, 3]] = -F[[1, 3, 4]];
```

```
In[250]:= F[[2, 2, 1]] = -F[[2, 1, 2]];
F[[3, 3, 1]] = -F[[3, 1, 3]]; F[[4, 4, 1]] = -F[[4, 1, 4]];
```

```
In[251]:= F[[2, 3, 1]] = -F[[2, 1, 3]]; F[[3, 4, 1]] = -F[[3, 1, 4]];
```

# Ricci scalar

$$R = -\frac{1}{4} M_{ad} M^{be} M^{cf} f^a{}_{bc} f^d{}_{ef} - \frac{1}{2} M^{cd} f^a{}_{bc} f^b{}_{ad} = -\frac{1}{4} A_1 - \frac{1}{2} A_2$$

```
In[252]:= A1 = Sum[M[[a, d]] Minv[[b, e]] Minv[[c, f]] F[[a, b, c]] F[[d, e, f]],
           {a, 1, 4}, {b, 1, 4}, {c, 1, 4}, {d, 1, 4}, {e, 1, 4}, {f, 1, 4}]
```

```
Out[252]= 2 m22 n11 n33 + 4 m23 n11 n34 - 2 m11 n342 + 2 m33 n11 n44 + 2 m11 n33 n44
```

```
In[253]:= A2 =
           Sum[Minv[[c, d]] F[[a, b, c]] F[[b, a, d]], {a, 1, 4}, {b, 1, 4}, {c, 1, 4}, {d, 1, 4}]
```

```
Out[253]= 2 n44
```

```
In[254]:= R = -1 / 4 A1 - 1 / 2 A2
```

```
Out[254]= -n44 +  $\frac{1}{4} \left( -2 m_{22} n_{11} n_{33} - 4 m_{23} n_{11} n_{34} + 2 m_{11} n_{34}^2 - 2 m_{33} n_{11} n_{44} - 2 m_{11} n_{33} n_{44} \right)$ 
```

```
In[255]:= Simplify[R]
```

```
Out[255]=  $\frac{1}{2} \left( -m_{22} n_{11} n_{33} - 2 m_{23} n_{11} n_{34} + m_{11} n_{34}^2 - 2 n_{44} - m_{33} n_{11} n_{44} - m_{11} n_{33} n_{44} \right)$ 
```

### Legend:

$n_{ab}$  indicates the ab\_element of the inverse metric. The same holds for  $m_{ab}$  which is an element of the regular metric. Empty spots (when present) are parentheses.