Suppose a unit charge (q) is brought from infinity to a source charge (with intensity Q)...some work is done in this process...what will it be?

Suppose the instantaneous distance of the unit charge from the source charge is r_i, then the force at distance r will be – $\frac{Qq}{4\pi\epsilon r_i^2}$

since work done = F times displacement, the work done for an infinitely small distance (with any value of instantaneous distance) will be $\frac{Qq}{4\pi\epsilon r_i^2}dr_i$ ...where dr is the infinitely small change in distance (or the infinitely small section of the x axis).

Integrate this expression from infinity to r (the constant at which you need to find the potential) and you will get the total work done -

$$\int_{\infty}^{r} \frac{Qq}{4\pi\epsilon r_i^2} dr_i$$

now r is the final distance...what this function is doing is returning the sum of infinitely small work done for till the distances r, now cause r is the final distance this will yield the summation of all the work done till r for each infinitely small distance dr.

Since q is 1 -

$\int_{\infty}^{r} \frac{Q}{4\pi\epsilon r_i^2} dr_i$ or $\int_{\infty}^{r} E\, dr_i$

v = $\frac{Q}{4\pi\epsilon r}$ .....Equation A.

So this expression is for the potential of a unit charge (generated form infinity) at a distance r from the source charge and relative to the source charge's frame of reference.

Now we're gonna derive a very useful expression (useful cause its used in many places and is popular)

We know $E=\frac{Q}{4\pi\epsilon r^2}$ ...This will be the E.F at a distance r from the charge Q.

Derive value of Q from the above.....

$$4E\pi\epsilon r^2 = Q$$

Substitute in equation A -

$$\frac{4E\pi\epsilon r^2}{4\pi\epsilon r}$$

$$v = Er$$