$$\left(\frac{2EI}{h^3}\begin{bmatrix} 2h^2 & 3h & h^2 & 0 \\ 3h & 12 & 0 & -3h \\ h^2 & 0 & 4h^2 & h^2 \\ 0 & -3h & h^2 & 2h^2 \end{bmatrix} - \frac{\rho A h \omega^2}{420}\begin{bmatrix} 4h^2 & -13h & -3h^2 & 0 \\ -13h & 312 & 0 & 13h \\ -3h^2 & 0 & 8h^2 & -3h^2 \\ 0 & 13h & -3h^2 & 4h^2 \end{bmatrix}\right) = \begin{bmatrix} u_2 \\ u_3 \\ u_4 \\ u_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{vmatrix} 2h^2 - 4h^2\lambda & 3h + 13h\lambda & h^2 + 3h^2\lambda & 0 \\ 3h + 13h\lambda & 12 - 312\lambda & 0 & -3h - 13h\lambda \\ h^2 + 3h^2\lambda & 0 & 4h^2 - 8h^2\lambda & h^2 + 3h^2\lambda \\ 0 & -3h - 13h\lambda & h^2 + 3h^2\lambda & 2h^2 - 4h^2\lambda \end{vmatrix}$$

$$\lambda = \frac{\omega^2 \rho A h^4}{840EI}$$

$$\omega = \sqrt{\frac{840\lambda EI}{\rho A h^4}}, \qquad \omega_{n,exact} = \frac{(n\pi)^2}{L^2}\sqrt{\frac{EI}{\rho A}} \qquad \text{where } n = 1, 2, 3, 4$$

MATLAB results:

$$det(A) = 36h^6 - 5232h^6\lambda + 41976h^6\lambda^2 - 51632h^6\lambda^3 + 12740h^6\lambda^4 = 4h^6(\lambda - 3)(7\lambda - 1)(455\lambda^2 - 414\lambda + 3)$$

setting the determinant equal to zero and solving for the eigenvalues and natural frequencies. E, I, ρ, A, and h are set equal to 1.

$$\lambda_1 = 0.007305, \qquad \omega_1 = 2.4771, \qquad \omega_{1,exact} = 9.8696$$

$$\lambda_2 = 0.142857, \qquad \omega_2 = 10.9545, \qquad \omega_{2,exact} = 39.4784$$

$$\lambda_3 = 0.902585, \qquad \omega_3 = 27.5349, \qquad \omega_{3,exact} = 88.8264$$

$$\lambda_4 = 3.000000, \qquad \omega_4 = 50.1996, \qquad \omega_{4,exact} = 157.9137$$