... continued

- Substituting (115) into (116) gives

S&J's form → $$\frac{df}{dt} = \frac{h}{r^2} + \frac{1}{eh}\left[e + \frac{r}{p}(\cos f + e)\right]\underline{r}^T\underline{a}_d - \frac{r}{h^2 e}(p+r)\sin f\,\underline{v}^T\underline{a}_d \qquad (117)$$

Battin's form → $$= \frac{h}{r^2} + \frac{r}{h^2 e}\left\{\left[\frac{eh}{r} + \frac{h}{p}(\cos f + e)\right]\underline{r}^T\underline{a}_d - (p+r)\sin f\,\underline{v}^T\underline{a}_d\right\}$$

- Next, substitute for position and velocity:

$$\underline{r}^T\underline{a}_d = \begin{bmatrix} r & 0 & 0 \end{bmatrix}\begin{bmatrix} a_r \\ a_\theta \\ a_h \end{bmatrix} = r a_r$$

$$\underline{v}^T\underline{a}_d = \begin{bmatrix} \frac{\mu}{h}e\sin f & \frac{\mu p}{rh} & 0 \end{bmatrix}\begin{bmatrix} a_r \\ a_\theta \\ a_h \end{bmatrix} = \frac{\mu}{h}e\sin f\, a_r + \frac{\mu p}{rh}a_\theta$$

$$\frac{df}{dt} = \frac{h}{r^2} + \frac{1}{eh}\left[e + \frac{r}{p}(\cos f + e)\right] r a_r - \frac{r}{h^2 e}(p+r)\sin f\left(\frac{\mu}{h}e\sin f\, a_r + \frac{\mu p}{rh}a_\theta\right)$$

$$= \frac{h}{r^2} + \frac{1}{eh}\left[\left(re + \frac{r^2}{p}(\cos f + e) - \frac{er\mu}{h^2}(p+r)\sin^2 f\right)a_r - \frac{\mu p}{h^2}(p+r)\sin f\, a_\theta\right]$$

$$= \frac{h}{r^2} + \frac{1}{eh}\left[\left(re + \frac{r^2}{p}\cos f + \frac{r^2 e}{p} - \frac{er\mu p}{h^2}\sin^2 f - \frac{er^2\mu}{h^2}\sin^2 f\right)a_r - (p+r)\sin f\, a_\theta\right]$$

$$= \frac{h}{r^2} + \frac{1}{eh}\left[\left(re\left(1 + \frac{r}{p}\right) + \frac{r^2}{p}\cos f - \sin^2 f\, er\left(1 + \frac{r\mu}{h^2}\right)\right)a_r - (p+r)\sin f\, a_\theta\right]$$

$$= \frac{h}{r^2} + \frac{1}{eh}\left[re\left(1 + \frac{r}{p}\right) + \frac{r^2}{p}\cos f - \sin^2 f\, er\left(1 + \frac{r}{p}\right)\right)a_r - (p+r)\sin f\, a_\theta\Big]$$

$$= \frac{h}{r^2} + \frac{1}{eh}\left[\left(\left(1 + \frac{r}{p}\right)re\left(1 - \sin^2 f\right) + \frac{r^2}{p}\cos f\right)a_r - (p+r)\sin f\, a_\theta\right]$$

$$= \frac{h}{r^2} + \frac{1}{eh}\left[p\cos f\, a_r - (p+r)\sin f\, a_\theta\right] \qquad (118)$$