ChatGPT

Step 3: Use Partial Fraction Decomposition

To find the inverse Laplace transform, decompose the fraction:

$$X(s) = \frac{37}{(s-6)^2(s^2+1)} = \frac{As+B}{s^2+1} + \frac{C}{s-6} + \frac{D}{(s-6)^2}.$$

Multiply both sides by $(s-6)^2(s^2+1)$ to find A, B, C, and D:

$$37 = (As+B)(s-6)^2 + C(s^2+1)(s-6) + D(s^2+1).$$

Step 4: Solve for the Coefficients

Setting up the equation:

$$37 = A(s^3 - 12s^2 + 36s - 6s + 36) + B(s^2 - 12s + 36) + C(s^3 - 6s + s - 6) + D(s^2 +$$

Collecting like terms, we equate coefficients of s^3, s^2, s, and constant term on both sides of the equation. Here we will solve for coefficients:

$$\begin{aligned} s^3 &: A + C = 0, \\ s^2 &: -12A + B + D = 0, \\ s &: 36A - 12B - 6C = 0, \\ 1 &: 36B + D = 37. \end{aligned}$$

From s^3 term: $A + C = 0 \implies C = -A.$

From s term: $36A - 12B - 6(-A) = 0 \implies 42A - 12B = 0 \implies 7A = 2B \implies B = \frac{7A}{2}$.

Substitute $B = \frac{7A}{2}$ and $C = -A$ into s^2 term:

$$-12A + \frac{7A}{2} + D = 0 \implies -24A + 7A + 2D = 0 \implies -17A + 2D = 0 \implies D = \frac{17}{}$$

Finally substitute $B = \frac{7A}{2}$ and $D = \frac{17A}{2}$ into constant term:

$$36\left(\frac{7A}{2}\right) + \frac{17A}{2} = 37 \implies 126A + 17A = 74 \implies 143A = 74 \implies A = \frac{74}{143} \implies$$

So,

$$X(s) = \frac{\frac{259}{143} + s \cdot \frac{74}{143}}{s^2 + 1} + \frac{\frac{17}{13}}{(s-6)^2} + \frac{-\frac{74}{143}}{s - 6}.$$

Step 5: Inverse Laplace Transform

Now take the inverse Laplace transform of each term separately:

$$\mathcal{L}^{-1}\left\{\frac{\frac{74}{143}s}{s^2+1} + \frac{\frac{259}{143}}{s^2+1}\right\} = \frac{74}{143}\cos(t) + \frac{259}{143}\sin(t),$$

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$$\mathcal{L}^{-1}\left\{\frac{17}{13} \cdot \frac{1}{(s-6)^2}\right\} = \frac{17}{13}te^{6t},$$

$$\mathcal{L}^{-1}\left\{-\frac{74}{143} \cdot \frac{1}{s-6}\right\} = -\frac{74}{143}e^{6t}.$$

Thus the final solution is:

$$x(t) = \frac{74}{143}\cos t + \frac{259}{143}\sin t + \frac{17}{13}te^{6t} - \frac{74}{143}e^{6t}$$