$$x = e^t$$

$$\frac{d(x)}{dt} = \frac{d}{dt}(e^t) \Rightarrow \frac{dx}{dt} = e^t$$

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = \frac{dy}{dx} \cdot e^t \Rightarrow \boldsymbol{\frac{dy}{dx} = \frac{1}{e^t} \cdot \frac{dy}{dt}}$$

$$\frac{d^2y}{dt^2} = \frac{d}{dt}\frac{dy}{dx} = \frac{d}{dx}\frac{dy}{dt} = \frac{d}{dx}\left(\frac{dy}{dx} \cdot e^t\right) = \frac{d}{dx}\left(\frac{dy}{dx} \cdot x\right) = \frac{d^2y}{dx^2} \cdot x + \frac{dy}{dx} \cdot 1$$

$$\boldsymbol{\frac{d^2y}{dx^2} = \left(\frac{d^2y}{dt^2} - \frac{dy}{dx}\right)\frac{1}{x}}$$

$$x^2 y'' - 4x\,y' + 6y = x^4 \sin(x)$$

$$x^2 \frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = x^4 \sin(x)$$

$$(e^t)^2 \frac{d^2y}{dx^2} - 4(e^t)\frac{dy}{dx} + 6y = (e^t)^4 \sin(e^t)$$

$$(e^t)^2 \left(\frac{d^2y}{dt^2} - \frac{dy}{dx}\right)\frac{1}{x} - 4(e^t)\left(\frac{1}{e^t} \cdot \frac{dy}{dt}\right) + 6y = (e^t)^4 \sin(e^t)$$

$$e^{2t} \left(\frac{d^2y}{dt^2} - \frac{dy}{dx}\right)\frac{1}{x} - 4 \cdot \frac{dy}{dt} + 6y = e^{4t} \sin(e^t)$$

$$e^{2t} \frac{d^2y}{dt^2}\frac{1}{x} - e^{2t} \frac{dy}{dx}\frac{1}{x} - 4 \cdot \frac{dy}{dt} + 6y = e^{4t} \sin(e^t)$$

$$e^{2t} \frac{d^2y}{dt^2} \cdot \frac{1}{x} - e^{2t} \cdot \frac{1}{e^t} \cdot \frac{dy}{dt} \cdot \frac{1}{x} - 4 \cdot \frac{dy}{dt} + 6y = e^{4t} \sin(e^t)$$

$$e^{2t} \frac{d^2y}{dt^2} \cdot \frac{1}{e^t} - e^t \cdot \frac{dy}{dt} \cdot \frac{1}{e^t} - 4 \cdot \frac{dy}{dt} + 6y = e^{4t} \sin(e^t)$$

$$e^t \frac{d^2y}{dt^2} - 5 \cdot \frac{dy}{dt} + 6y = e^{4t} \sin(e^t)$$

$$\frac{d^2y}{dt^2} - 5e^{-t} \cdot \frac{dy}{dt} + 6e^{-t} y = e^{3t} \sin(e^t)$$