

Problem: The system I am trying to replicate is a tube initially filled with liquid A. The tube has a radius $r = R$. The liquid is displaced by liquid B entering the tube under fully developed laminar flow. This displaces all the fluid A except for a small volume present at the outer radius (i.e. at the tube wall) that remains because at the outer radius flow velocity is very low and diffusion dominates. I would like to examine how the remaining solute in liquid A diffuses from the outer walls of a tube into the bulk, when the flow stops. I am of course looking for a solution to Fick's second law, but I am a real novice when it comes to setting the boundary and initial conditions.

The concentration is homogenous along the direction of flow and the problem is reduced to 2 Dimensions (i.e. a circle with diffusion towards center). The volume of A is very small ($\delta \ll R$) compared to the bulk volume of B. There is conservation of mass and the total amount of solute is constant.

Initially the concentration of solute in A = C_1 and in B = 0

Initial conditions

$$R - \delta < r < R \quad C = C_1$$

$$r < R - \delta \quad C = 0$$

Boundary conditions

$$r = 0 \quad \frac{\partial C_i}{\partial r} = 0$$

$$\text{constant mass} = \int_{-\infty}^{\infty} C(r, t) 2\pi r \, dr$$

