Assignment #4

Problem #1

General Definitions:
a m^3/day = the flow rate to the filter = the flow rate from the filter
2 a m^3/day = the total flow rate from Lake B to Lake A
1,000,000 m^3 = the volume of Lake A
500,000 m^3 = the volume of Lake B
1/10 = the percentage of Dioxin returned from the filter
y(x) = the amount of Dioxin in Lake A @ x
z(x) = the amount of Dioxin in Lake B @ x
30,000 m^3/day = the flow rate of Stream 1 = the flow rate of Stream 2 = the flow rate of Stream 3

Dioxin per day added to Lake A:
(1/10) ((2 a)/(1,000,000)) z(x)

Dioxin per day from Lake A to Lake B:
(30,000 + a) (1/1,000,000) y(x)

Dioxin per day removed from Lake B:
((2 a)/(1,000,000) + (30,000)/(50,000)) z(x)

Change in concentration of Dioxin in Lake A for change in time:
y(x)' = (1/10) ((2 a)/(1,000,000)) z(x) - (30,000 + a) (1/1,000,000) y(x)

Change in concentration of Dioxin in Lake B for change in time:
z(x)' = (30,000 + a) (1/1,000,000) y(x) - ((2 a)/(1,000,000) + (30,000)/(500,000)) z(x)

Initial Conditions:
y(0) = ((1/10E11)*110000)*1000000 = 0.11
z(0) = ((1/10E11)*500000)*1600000 = 0.8

$$DEQ1 := y' = \left(\left(\frac{1}{10}\right)\cdot\left(\frac{(2\cdot a)}{1000000}\right)\cdot z\right) - \left(\left(\frac{(30000+a)}{1000000}\right)\cdot y\right),\ z' = \left(\frac{(30000+a)}{1000000}\cdot y\right) - \left(\left(\left(\frac{2\cdot a}{1000000}\right) + \left(\frac{30000}{500000}\right)\right)\cdot z\right);$$

$$\frac{d}{dx}y(x) = \frac{1}{5000000}\,a\,z(x) - \frac{1}{1000000}\,(30000+a)\,y(x),\ \frac{d}{dx}z(x) = \frac{1}{1000000}\,(30000+a)\,y(x) - \left(\frac{1}{500000}\,a + \frac{3}{50}\right)z(x) \tag{1}$$

$$ICS1 := y(0) = 0.11, z(0) = 0.8;$$

$$y(0) = 0.11, z(0) = 0.8 \tag{2}$$

$$DSoln := dsolve([DEQ1, ICS1]);$$

$$\left\{ y(x) = \frac{1}{10}\,\frac{1}{30000+a}\left(\frac{500\left(-29\sqrt{22500000000 + 2100000\,a + 45\,a^2} + 6000000 + 360\,a\right)e^{\left(-\frac{9}{200} - \frac{3}{2000000}\,a + \frac{1}{10000000}\sqrt{22500000000 + (2100000+45\,a)\,a}\right)x}}{3\,a + 50000} \right. \tag{3}$$

$$+ \frac{1}{60}\,\frac{\left(-29\sqrt{22500000000 + 2100000\,a + 45\,a^2} + 6000000 + 360\,a\right)a\,e^{\left(-\frac{9}{200} - \frac{3}{2000000}\,a + \frac{1}{10000000}\sqrt{22500000000 + (2100000+45\,a)\,a}\right)x}}{3\,a + 50000}$$

$$+ \frac{1}{300}\,\frac{\left(-29\sqrt{22500000000 + 2100000\,a + 45\,a^2} + 6000000 + 360\,a\right)\sqrt{22500000000 + (2100000+45\,a)\,a}\,e^{\left(-\frac{9}{200} - \frac{3}{2000000}\,a + \frac{1}{10000000}\sqrt{22500000000 + (2100000+45\,a)\,a}\right)x}}{3\,a + 50000}$$

$$+ \frac{500\left(29\sqrt{22500000000 + 2100000\,a + 45\,a^2} + 6000000 + 360\,a\right)e^{\left(-\frac{9}{200} - \frac{3}{2000000}\,a - \frac{1}{10000000}\sqrt{22500000000 + (2100000+45\,a)\,a}\right)x}}{3\,a + 50000}$$

$$+ \frac{1}{60}\,\frac{\left(29\sqrt{22500000000 + 2100000\,a + 45\,a^2} + 6000000 + 360\,a\right)a\,e^{\left(-\frac{9}{200} - \frac{3}{2000000}\,a - \frac{1}{10000000}\sqrt{22500000000 + (2100000+45\,a)\,a}\right)x}}{3\,a + 50000}$$

$$\left. - \frac{1}{300}\,\frac{\left(29\sqrt{22500000000 + 2100000\,a + 45\,a^2} + 6000000 + 360\,a\right)\sqrt{22500000000 + (2100000+45\,a)\,a}\,e^{\left(-\frac{9}{200} - \frac{3}{2000000}\,a - \frac{1}{10000000}\sqrt{22500000000 + (2100000+45\,a)\,a}\right)x}}{3\,a + 50000} \right),$$

$$z(x) = \frac{1}{300}\,\frac{\left(-29\sqrt{22500000000 + 2100000\,a + 45\,a^2} + 6000000 + 360\,a\right)e^{\left(-\frac{9}{200} - \frac{3}{2000000}\,a + \frac{1}{10000000}\sqrt{22500000000 + (2100000+45\,a)\,a}\right)x}}{3\,a + 50000}$$

$$\left. + \frac{1}{300}\,\frac{\left(29\sqrt{22500000000 + 2100000\,a + 45\,a^2} + 6000000 + 360\,a\right)e^{\left(-\frac{9}{200} - \frac{3}{2000000}\,a - \frac{1}{10000000}\sqrt{22500000000 + (2100000+45\,a)\,a}\right)x}}{3\,a + 50000} \right\}$$

select entry 2 →

$$z(x) = \frac{1}{300}\,\frac{\left(-29\sqrt{22500000000 + 2100000\,a + 45\,a^2} + 6000000 + 360\,a\right)e^{\left(-\frac{9}{200} - \frac{3}{2000000}\,a + \frac{1}{10000000}\sqrt{22500000000 + (2100000+45\,a)\,a}\right)x}}{3\,a + 50000} \tag{4}$$

$$+ \frac{1}{300}\,\frac{\left(29\sqrt{22500000000 + 2100000\,a + 45\,a^2} + 6000000 + 360\,a\right)e^{\left(-\frac{9}{200} - \frac{3}{2000000}\,a - \frac{1}{10000000}\sqrt{22500000000 + (2100000+45\,a)\,a}\right)x}}{3\,a + 50000}$$

assign as function →

$$z \tag{5}$$

select entry 1 →

$$y(x) = \frac{1}{10}\,\frac{1}{30000+a}\left(\frac{500\left(-29\sqrt{22500000000+2100000\,a+45\,a^2}+6000000+360\,a\right)e^{\left(-\frac{9}{200}-\frac{3}{2000000}a+\frac{1}{10000000}\sqrt{22500000000+(2100000+45\,a)\,a}\right)x}}{3\,a+50000}\right. \tag{6}$$

$$+\frac{1}{60}\,\frac{\left(-29\sqrt{22500000000+2100000\,a+45\,a^2}+6000000+360\,a\right)a\,e^{\left(-\frac{9}{200}-\frac{3}{2000000}a+\frac{1}{10000000}\sqrt{22500000000+(2100000+45\,a)\,a}\right)x}}{3\,a+50000}$$

$$+\frac{1}{300}\,\frac{\left(-29\sqrt{22500000000+2100000\,a+45\,a^2}+6000000+360\,a\right)\sqrt{22500000000+(2100000+45\,a)\,a}\,e^{\left(-\frac{9}{200}-\frac{3}{2000000}a+\frac{1}{10000000}\sqrt{22500000000+(2100000+45\,a)\,a}\right)x}}{3\,a+50000}$$

$$+\frac{500\left(29\sqrt{22500000000+2100000\,a+45\,a^2}+6000000+360\,a\right)e^{\left(-\frac{9}{200}-\frac{3}{2000000}a-\frac{1}{10000000}\sqrt{22500000000+(2100000+45\,a)\,a}\right)x}}{3\,a+50000}$$

$$+\frac{1}{60}\,\frac{\left(29\sqrt{22500000000+2100000\,a+45\,a^2}+6000000+360\,a\right)a\,e^{\left(-\frac{9}{200}-\frac{3}{2000000}a-\frac{1}{10000000}\sqrt{22500000000+(2100000+45\,a)\,a}\right)x}}{3\,a+50000}$$

$$\left.-\frac{1}{300}\,\frac{\left(29\sqrt{22500000000+2100000\,a+45\,a^2}+6000000+360\,a\right)\sqrt{22500000000+(2100000+45\,a)\,a}\,e^{\left(-\frac{9}{200}-\frac{3}{2000000}a-\frac{1}{10000000}\sqrt{22500000000+(2100000+45\,a)\,a}\right)x}}{3\,a+50000}\right)$$

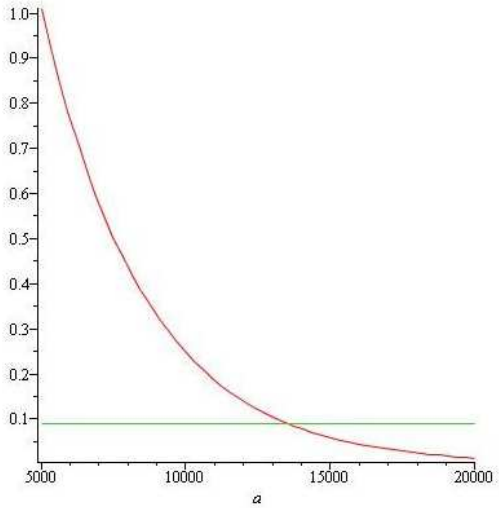
assign as function →

$$y \tag{7}$$

$f := a \to \frac{2}{1000000}\cdot z(365)\cdot 1000000000000;$

$$a \to 2000000\,z(365) \tag{8}$$

$plot([f(a), 0.09], a = 5000..20000);$



$fsolve(f(a) = 0.09, a);$

$$-11400.69354 \tag{9}$$