

proposition)

$$\forall \ 0 < a < 1, \text{ there is } b \text{ satisfying } \zeta(s) = \sum_{k=1}^{\infty} k^{-s} = 0 (s = a + bi)$$

proof)

When $N \neq A^B (1 < B) (A, B \in \mathbb{N})$. Let define N to independent number, i-th independent number is D_i

For example, 6 is D_i , $8 = 2^3$ isn't an independent number.

Zeta function is

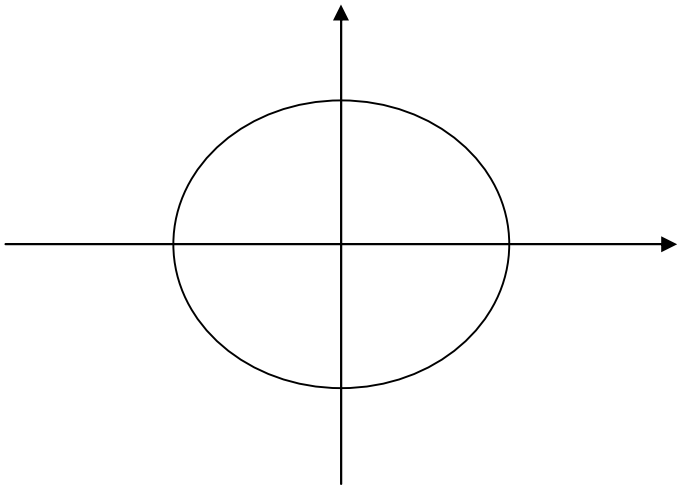
$$\zeta(s) = \sum_{k=1}^{\infty} k^{-s} = 1 + \sum_{i=1}^{\infty} \frac{\frac{1}{D_i^s}}{1 - \frac{1}{D_i^s}}$$

$$\text{and, } k^{-s} = \frac{1}{k^a} \cdot e^{\left\{ \frac{\ln n \cdot b}{2\pi} \right\} \cdot 2\pi i} \left(\left\{ -\frac{\ln n \cdot b}{2\pi} \right\} \text{ is decimal part of } -\frac{\ln n \cdot b}{2\pi} \right)$$

for fixed a , some b satisfies

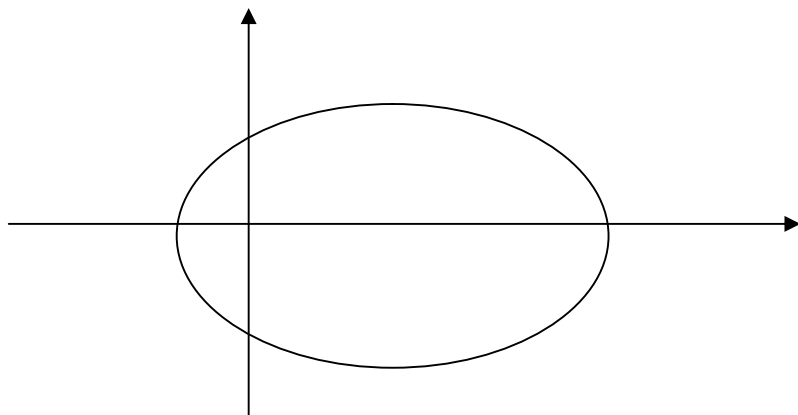
$$\text{if we can select } \left\{ -\frac{\ln D_i \cdot b}{2\pi} \right\} \in [0, 1)$$

possible $\frac{1}{D_i^s}$'s trace is

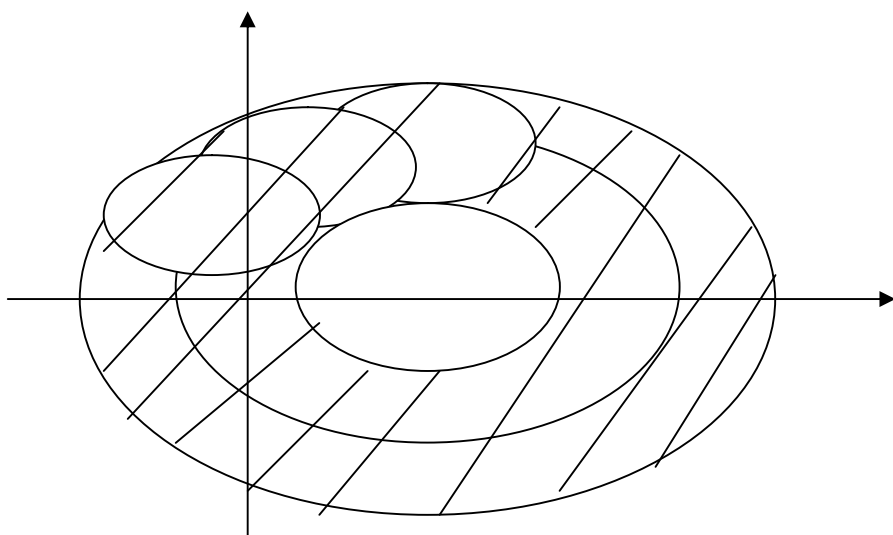


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$\frac{\frac{1}{D_i^s}}{1 - \frac{1}{D_i^s}}$'s trace is



$\frac{\frac{1}{D_1^s}}{1 - \frac{1}{D_1^s}} + \frac{\frac{1}{D_2^s}}{1 - \frac{1}{D_2^s}}$'s trace is



When proving we can select $\left\{-\frac{\ln D_i \cdot b}{2\pi}\right\} \in [0,1)$, From that $\lim_{k \rightarrow \infty} \left(\sum_{i=1}^k \frac{1}{P_i}\right) = \infty$, we know

$$\sum_{i=1}^{\infty} \frac{\frac{1}{D_i^s}}{1 - \frac{1}{D_i^s}} \text{ is } \forall \text{ complex number for proper } b$$

$$\text{Hence, } \zeta(s) = \sum_{k=1}^{\infty} k^{-s} = 1 + \sum_{i=1}^{\infty} \frac{\frac{1}{D_i^s}}{1 - \frac{1}{D_i^s}} = \forall \text{ complex number}$$

$$\pm \frac{1}{2} \pm \frac{1}{3} \pm \frac{1}{5} \pm \dots \pm \frac{1}{P_m} = \frac{P_{m_n}}{\prod_{i=1}^m P_i} \quad (P_m - \text{prime is not divided by under } P_m, P_{m_n} \text{ is always } P_m - \text{prime})$$

$$\pm \frac{1}{2} \pm \frac{1}{3} \pm \frac{1}{5} \pm \dots \pm \frac{1}{P_m} \text{ Is continuous for large } P_m, \text{ we can find } (P_{m_2} : P_{m_3} : P_{m_5} : \dots)$$

satisfying

$$(P_{m_2} : P_{m_3} : P_{m_5} : \dots) = \left(\frac{1}{\ln 2} : \frac{1}{\ln 3} : \frac{1}{\ln 5} : \dots \right)$$

$$\text{And } (P_{m_2} : P_{m_3} : P_{m_5} : \dots) = \lim_{n \rightarrow \infty} (2n \cdot P_{m_2} + 1 : 2n \cdot P_{m_3} + 1 : 2n \cdot P_{m_5} + 1 : \dots)$$

If $2n \cdot P_{m_n} + 1$ s are all primes,

$$\left\{ \frac{k}{2n \cdot P_{m_2} + 1} \right\}, \left\{ \frac{k}{2n \cdot P_{m_3} + 1} \right\}, \left\{ \frac{k}{2n \cdot P_{m_5} + 1} \right\}, \dots \text{ contains every case}$$

$$\frac{0, 1, 2, 3, 4, \dots}{2n \cdot P_{m_2} + 1}, \frac{0, 1, 2, 3, 4, \dots}{2n \cdot P_{m_3} + 1}, \frac{0, 1, 2, 3, 4, \dots}{2n \cdot P_{m_5} + 1}, \dots$$

$$\text{And we can find } \left(\left\{ \frac{k}{2n \cdot P_{m_2} + 1} \right\} : \left\{ \frac{k}{2n \cdot P_{m_3} + 1} \right\} : \left\{ \frac{k}{2n \cdot P_{m_5} + 1} \right\} : \dots \right) = (\ln 2 : \ln 3 : \ln 5 : \dots)$$

excluding j -unit $f_i(n) = a_i n + b_i (a_i, n, b_i \in \mathbb{Z})$ contains Composite number deservedly
(For example $f_1(n) = n, f_2(n) = n + 1, f_3(n) = n + 2$)

For $\forall i$, there exist n_k that $f_i(n_k)$ is a P_k -prime, and

$$f_i(n_k) \equiv f_i(n_k + n \cdot \prod_{i=1}^k P_i) \pmod{P_k}, \text{ hence } f_i(n_k + n \cdot \prod_{i=1}^k P_i) \text{ is also a } P_k\text{-prime}$$

And P_k -prime under P_k^2 is a prime.

Define $\frac{1}{s(k)}$ to ratio of P_k -prime In $1, 2, 3, \dots, \prod_{i=1}^k P_i$,

For K satisfying $\prod_{i=1}^k P_i < K$

$$\text{ratio of } P_k\text{-prime} \leq \frac{2}{s(k)+1} \text{ In } 1, 2, 3, \dots, K$$

Less than $(\text{Max}(f_i(n)))^2, \text{Max}(f_i(n))$ -unit $f_i(n)$ s are P_k -prime, only have a factor

F that satisfies $P_{k+1} \leq F \leq \text{Max}(f_i(n))$

If $\prod_{i=1}^k P_i < \text{Max}(f_i(n))$, ratio of P_k -prime $\leq \frac{2}{s(k)+1}$ In $1, 2, 3, \dots, \text{Max}(f_i(n))$

When arrange only factors of $1, 2, 3, \dots, \text{Max}(f_i(n))$,

• 2 • 2 • 2 • 2 • ...
 • • 3 • • 3 • • 3 ...
 • • • • 5 • • • • ...
 • • • • • • 7 • • ...
 • • • • • • • • • ...

When $\forall P_i$'s line moving horizontally, number of Composite number doesn't increase

from $x \equiv a \pmod{P_a}, x \equiv b \pmod{P_b}$'s minimum value $< P_a \cdot P_b$

(and from this, we can know there's a prime $N \sim 2N$ easily)

Shortly, we can find sequence of numbers lenth is longer than P_m that $P_m < \text{Max}(f_i(n))$

And under P_m^2 , and every numbers take factors only P_k -prime $\simeq P_m$ but that's

$$\text{maximum ratio is under } \frac{2j}{s(k)+1}$$

Hence $Max(f_i(n))$ -unit $f_i(n)$ s under $(Max(f_i(n)))^2$ contains Composite numbers under ratio $\frac{2}{s(k)+1}$, when it's under $\frac{1}{j}$, j -unit $f_i(n)=a_i n+b_i$ can't all numbers are Composite number, Hence there exist n satisfying for $\forall i$, $f_i(n)=a_i n+b_i$ is a prime. It's so easy to know $\lim_{k \rightarrow \infty} \frac{1}{s(k)} = 0$, hence, for $\forall j$, we can find k satisfying $\frac{1}{j} > \frac{2}{s(k)+1}$, there exist n satisfying for $\forall i$, $f_i(n)=a_i n+b_i$ is a prime.

When $\frac{1}{j} = \frac{1}{2} > \frac{2}{s(k)+1}$, if $\left(\prod_{i=1}^k P_i\right)^2 < n$ here some conjecture's are proved

We can always find $f_1(n)=n, f_2(n)=2N-n$ are simultaneously prime (Goldbach's conjecture)

$f_1(n)=n, f_2(n)=2n+1$, Sophie Germain conjecture

$f_1(n)=n, f_2(n)=n+2$, twin prime conjecture

When $\frac{1}{j} = \frac{1}{d} > \frac{2}{s(k)+1}$, if $\left(\prod_{i=1}^k P_i\right)^2 < n$

$$f_1(n)=n+1, f_2(n)=2n+1, f_3(n)=3n+1, \dots, f_d(n)=d \cdot n+1$$

Hence we can find infinitely many n that $2n \cdot P_{m_2} + 1, 2n \cdot P_{m_3} + 1, 2n \cdot P_{m_5} + 1, \dots$ are simultaneously prime,

$$(P_{m_2} : P_{m_3} : P_{m_5} : \dots) = \lim_{n \rightarrow \infty} (2n \cdot P_{m_2} + 1 : 2n \cdot P_{m_3} + 1 : 2n \cdot P_{m_5} + 1 : \dots)$$

Hence, $\forall 0 < a < 1$, there is b satisfying $\zeta(s) = \sum_{k=1}^{\infty} k^{-s} = 0 (s = a + bi)$

More over, we can find b that $\zeta(s) = \sum_{k=1}^{\infty} k^{-s} = 1 + \sum_{i=1}^{\infty} \frac{\frac{1}{D_i^s}}{1 - \frac{1}{D_i^s}} = \text{any complex number}$

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