Division Algorithm

October 4, 2014

1 Algorithm

To figure out $\frac{a_0}{b}$, where $b \neq 1$:

Let $a_0/b = c$.

Step 1)

$$\frac{a_0}{b} = a_0 \left(\frac{1}{b}\right)$$

Step 2)

$$\frac{1}{b} = \frac{2}{2b} = \cdots = \frac{2^{n_1}}{2^{n_1} b}, \quad 2^{n_1} b \leq a < 2^{n_1+1} b$$

Step 3)

Let $a_1 = a - 2^{n_1} b$. Then,

$$\frac{a}{b} = \frac{a - 2^{n_1} b}{b} + 2^{n_1}$$

Step 4)

Repeat steps 1 and 2 with a_1/b. i.e,

$$a_k = a_{k-1} - 2^{n_k} b$$

So,

$$\frac{a_0}{b} = \frac{a_k}{b} + \sum_{i=1}^{k} 2^{n_i} = \frac{a_0 - b \sum_{i=1}^{k} 2^{n_i}}{b} + \sum_{i=1}^{k} 2^{n_i}.$$

Repeat until $0 \leq a_k \leq b$.

The goal is to get a_k/b to either 0, 1 or where $a_k < b$, in which case, a_k is the remainder.

1.1 Example

$$\frac{50}{7} = 50\left(\frac{1}{7}\right)$$

$$\frac{1}{7} = \frac{2}{2*7} = \frac{2^2}{2^2*7}, \quad 2^2*7 = 28 \leq 50 \leq 2^3*7 = 56$$

So $2^{n_1} = 2^2$.

$$a_1 = 50 - 2^2*7 = 22$$

$$\frac{22}{7} = 22\left(\frac{1}{7}\right)$$

$$\frac{1}{7} = \frac{2}{2*7}, \quad 2*7 = 14 \leq 22 \leq 2^2*7 = 28$$

So $2^{n_2} = 2$.

$$a_2 = 22 - 14 = 8$$

$$\frac{8}{7} = 8\left(\frac{1}{7}\right)$$

$$\frac{1}{7} = \frac{2^0}{2^0*7}$$

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So $2^{n_3} = 2^0$.

$$a_3 = 8 - 7 = 1$$

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Thus,

$$\frac{50}{7} = \frac{1}{7} + 1 + 2 + 2^2 = 7 + \frac{1}{7}$$