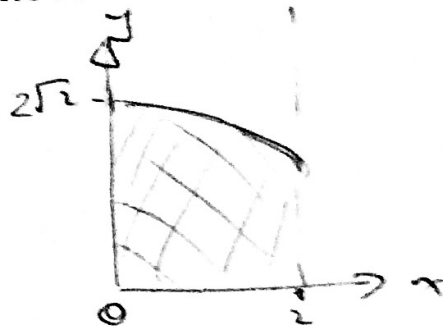


1. (17 marks) Use polar coordinates to find the area of the region in the first quadrant, which is bounded by the x -axis, the line $x = 2$, and the circular arc $x^2 + y^2 = 8$.

HINT: For a given choice of θ , what is the range for r ?

Use the formula $\int \frac{du}{\cos^2 u} = \tan u + C$.

$$x^2 + y^2 = (\sqrt{8})^2$$

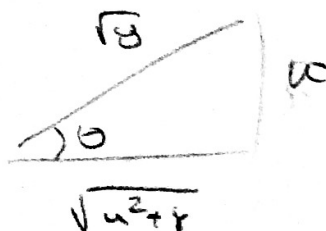


$$= \int_0^2 \int_0^{\sqrt{8-x^2}} dy dx$$

$$= \int_0^2 \sqrt{8-x^2} dx$$

$$u = \sqrt{8} \sin \theta$$

$$du = \sqrt{8} \cos \theta d\theta$$



$$= \int (\sqrt{8}) \sqrt{1 - \sin^2 \theta} (\sqrt{8} \cos \theta) d\theta$$

$$= 8 \int \cos^2 \theta d\theta$$

$$= \frac{8}{2} \int (1 + \cos 2\theta) d\theta$$

$$= 4 \left(\theta + \frac{\sin 2\theta}{2} \right) \Big|_0^{\frac{\pi}{4}}$$

$$= 4 \left(\frac{\pi}{4} + \frac{\sin(\frac{\pi}{2})}{2} - \left(0 + \frac{\sin(0)}{2} \right) \right)$$

$$= 4 \left(\frac{\pi}{4} + \frac{1}{2} \right)$$

$$= \boxed{\pi + 2}$$

$$\frac{2}{\sqrt{8}} = \sin \theta$$

$$\frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \sin \theta$$

$$\theta = \frac{\pi}{4}$$

$$0 = \sin \theta$$

$$\theta = 0$$