

To see this, let us first consider a system with total energy E that consists of two weakly interacting subsystems. In this context, *weakly interacting* means that the subsystems can exchange energy but that we can write the total energy of the system as the sum of the energies E_1 and E_2 of the subsystems. There are many ways in which we can distribute the total energy over the two subsystems such that $E_1 + E_2 = E$. For a given choice of E_1 , the total number of degenerate states of the system is $\Omega_1(E_1) \times \Omega_2(E_2)$. Note that the total number of states is not the sum but the product of the number of states in the individual systems. In what follows, it is convenient to have a measure of the degeneracy of the subsystems that is additive. A logical choice is to take the (natural) logarithm of the degeneracy. Hence:

$$\ln \Omega(E_1, E - E_1) = \ln \Omega_1(E_1) + \ln \Omega_2(E - E_1). \quad (2.1.1)$$

We assume that subsystems 1 and 2 can exchange energy. What is the most likely distribution of the energy? We know that *every* energy state of the total system is equally likely. But the number of eigenstates that correspond to a given distribution of the energy over the subsystems depends very strongly on the value of E_1 . We wish to know the most likely value of E_1 , that is, the one that maximizes $\ln \Omega(E_1, E - E_1)$. The condition for this maximum is that

$$\left(\frac{\partial \ln \Omega(E_1, E - E_1)}{\partial E_1} \right)_{N, V, E} = 0 \quad (2.1.2)$$

or, in other words,

$$\left(\frac{\partial \ln \Omega_1(E_1)}{\partial E_1} \right)_{N_1, V_1} = \left(\frac{\partial \ln \Omega_2(E_2)}{\partial E_2} \right)_{N_2, V_2}. \quad (2.1.3)$$

We introduce the shorthand notation

$$\beta(E, V, N) \equiv \left(\frac{\partial \ln \Omega(E, V, N)}{\partial E} \right)_{N, V}. \quad (2.1.4)$$

With this definition, we can write equation (2.1.3) as

$$\beta(E_1, V_1, N_1) = \beta(E_2, V_2, N_2). \quad (2.1.5)$$

Clearly, if initially we put all energy in system 1 (say), there will be energy transfer from system 1 to system 2 until equation (2.1.3) is satisfied. From that moment on, no net energy flows from one subsystem to the other, and we say that the two subsystems are in (thermal) equilibrium. When this