Comparing eqns. [2.10] and [2.11] it is seen that the circumferential stress is twice the axial stress. Figure 2.12(*d*) shows a small element of the shell subjected to the axial and circumferential (hoop) stresses.

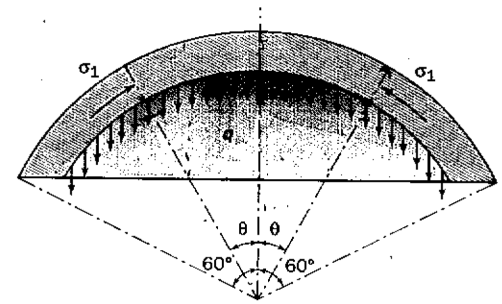
Example 2.2

A concrete dome is 250 mm thick, has a radius of 30 m and subtends an angle of 120° at the support ring. Calculate the stresses at the supports due to self-weight. The density for contrete is 2.3 Mg/m³. The dome is illustrated in Fig. 2.13.

$F = \sigma \cdot A$

radial component $= \sigma_1 \sin\theta$

Fig. 2.13



Consider the vertical equilibrium of a circular segment containing an angle 2θ; then if q is the force per unit area due to weight of concrete,

$$(2\pi r \sin\theta \times t\sigma_1 \sin\theta) + 2\pi r(r - r\cos\theta)q = 0$$

$$\sigma_1 = -\frac{qr}{t}\frac{(1-\cos\theta)}{\sin^2\theta} = -\frac{qr}{t}\frac{1}{(1+\cos\theta)}$$

Since self-weight, unlike applied pressure, does not act radially everywhere, eqn. [2.8] has to be modified to take account of the radially resolved component of weight, giving

$$\frac{\sigma_1}{r_1} + \frac{\sigma_2}{r_2} = -\frac{q}{t}\cos\theta$$

Substituting for σ_1,

$$\sigma_2 = \frac{qr}{t}\left(\frac{1}{1+\cos\theta} - \cos\theta\right)$$

The stresses at the supports are obtained by putting $\theta = 60°$ and $q = 2.3 \times 9.81 \times 250/1000 = 5.65\,\mathrm{kN/m^2}$:

$$\sigma_1 = -\frac{5.65 \times 30}{0.25} \times \frac{1}{1\frac{1}{2}} = -452\,\mathrm{kN/m^2}$$

$$\sigma_2 = \frac{5.65 \times 30}{0.25}\left(\tfrac{2}{3} - \tfrac{1}{2}\right) = 113\,\mathrm{kN/m^2}$$