

The Galilei transformations are

$$x'^i = x^i - v^i t, \quad (1)$$

$$t' = t, \quad (2)$$

where $i = 1, 2, 3$. The electric field E^i is a 3-vector, so under arbitrary coordinate transformations it transforms as

$$E'^i = \frac{\partial x'^i}{\partial x^j} E^j. \quad (3)$$

The coordinate transformation (1) gives

$$\frac{\partial x'^i}{\partial x^j} = \delta_j^i, \quad (4)$$

so Eq. (3) gives

$$E'^i = E^i. \quad (5)$$

Q.E.D.