

Lets say the I have two independent variables X and Y such that

$$X \in A, Y \in B \quad (1)$$

with

$$X = f_X(x), Y = f_Y(y) \quad (2)$$

we define a random Variable Z such that

$$Z = g(X, Y) \quad (3)$$

that is to say the Z is some function of random variable X and Y. Now by the law of total probability

$$\begin{aligned} F_Z(z) &= P(Z \leq z) \\ &= \int_B P(Z \leq z | Y = y) f_Y(y) dy \\ &= \int_B P(g(X, Y) \leq z | Y = y) f_Y(y) dy \\ &= \int_B P(X \leq h(y, z) | Y = y) f_Y(y) dy \end{aligned} \quad (4)$$

where  $h(y, z)$  is the algebraic result of solving  $g(X, Y)$  for X. This gives

$$F_Z(z) = \int_B F_{X|Y}(h(y, z), y) f_Y(y) dy \quad (5)$$

now since X and Y are independent variables

$$\begin{aligned} F_{X|Y}(x, y) &= F_X(h(y, z), y) \\ &= P(x \leq h(y, z) | Y = y) \\ &= \int_B \int_{-\infty}^{h(y, z)} f_X(x) f_Y(y) dx dy \\ &= \int_B F_X(h(y, z)) f_Y(y) dy \end{aligned} \quad (6)$$

Now if I sub eq. 6 into eq. 5 you get

$$\int_B \int_B F_X(h(y, z)) f_Y(y) dy f_Y(y) dy \quad (7)$$

which just does not seem right so I am wondering where I went wrong thanks