Referred/Deferred FD Assessment Sheet

Q.1. Discuss the different ways in which the first derivative :

$$\frac{\partial u}{\partial x}$$

can be expressed in terms of finite differences on a discrete grid. Your answer should include a discussion of the truncation error derived using a Taylor expansion as shown in the lectures. How might you apply this to finding an expression for the term

$$u\frac{\partial u}{\partial x} \qquad ?$$

Q.2. An equation often encountered in the study of numerical algorithms is the so-called 1-d Burgers equation;

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$$

This equation originally arose in the study of the motion of dislocations in metals, but it is often used in studying the behaviour of numerical algorithms for computing shock waves. We want to solve this equation for the following problem; on a domain $0 \leq x \leq 2\pi$, the initial conditions are given by

$$u(x, t = 0) = \sin x$$

and the boundary conditions are $u(x = 0, t) = u(x = 2\pi, t) = 0$;

a. Construct an explicit finite difference algorithm to solve the Burger's equation. Use a centered differencing scheme for the spatial derivative.

b. Construct in Excel a spreadsheet to solve this problem on a mesh with 20 nodes. Use a Courant number less than 0.25.

c. How does the solution evolve from $t = 0$ to $t = 2.0$? What is causing the observed behaviour, and why might it be interesting to someone studying shocks? (Hint; do some reading around the subject of Burger's equation).

Q.3. B (M.Eng students only) :

Use von Neumann stability analysis (as described in the lectures) to investigate the stability of your numerical scheme for Burger's equation.