Magnetic and Electric Field of a solenoid

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$$\nabla E = 0 \tag{1}$$

$$\nabla B = 0 \tag{2}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{3}$$

$$\nabla \times B = \mu\epsilon\frac{\partial E}{\partial t} + \mu\sigma E \tag{4}$$

The modified wave equations are then:

$$\nabla^2 E = \mu\epsilon\frac{\partial^2 E}{\partial t^2} + \mu\sigma\frac{\partial E}{\partial t} \tag{5}$$

$$\nabla^2 B = \mu\epsilon\frac{\partial^2 B}{\partial t^2} + \mu\sigma\frac{\partial B}{\partial t} \tag{6}$$

The magnetic field inside the sample will be homogenous and sinusoidally varying.

$$B_z(t) = B_0 cos(\omega t) \tag{7}$$

Plugging this into equation (25):

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix} = -\frac{\partial B}{\partial t} = \omega B_0 sin(\omega t)\hat{k} \tag{8}$$

or equivalently:

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = 0 \tag{9}$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = 0 \tag{10}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = \omega B_0 sin(\omega t) \tag{11}$$

Using equation (26)

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 0 & B_0 cos(\omega t) \end{vmatrix} = \mu\epsilon\frac{\partial E}{\partial t} + \mu\sigma E \tag{12}$$

or equivalently:

$$\mu\epsilon\frac{\partial E}{\partial t} = -\mu\sigma E \tag{13}$$