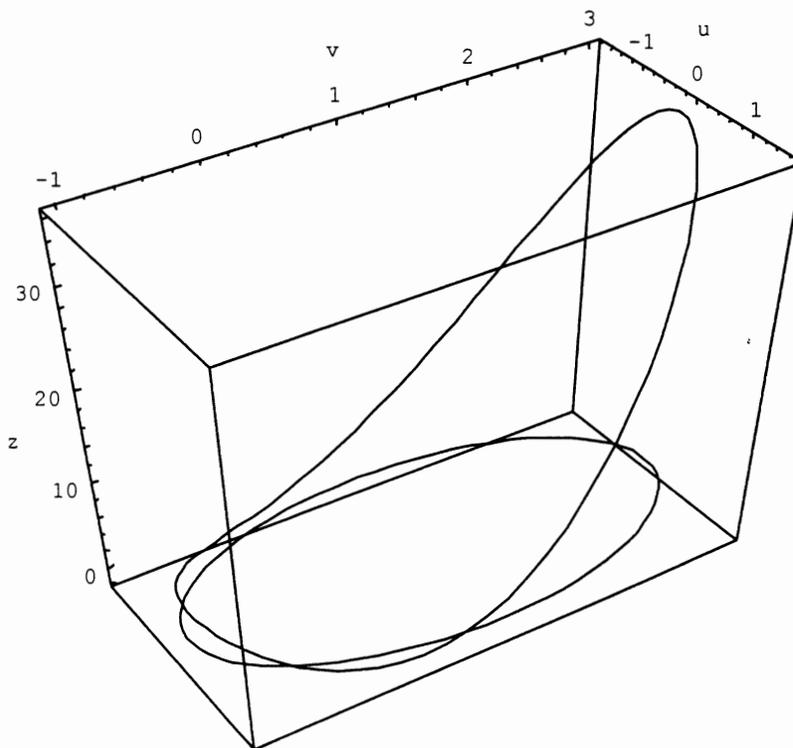


5.12 Constrained optimization over an ellipse

Description Use each of the following methods to determine the maximum and minimum values of the function $4x^2 + 4y^2 - 8xy - x - y$, subject to the constraint $7x^2 + 7y^2 + 2xy - 6x + 6y = 9$, and compare the time needed for these methods.



Optimization over an ellipse

Instructions (a) Apply directly the method of Lagrange multipliers.

(b) Use a linear change of coordinates $x = au + bv$, $y = cu + dv$ to obtain a standard form of the elliptic constraint and then apply the method of Lagrange multipliers with respect to the new variables u and v .

(c) Determine a parametrization of the elliptic constraint over the interval $[0, 2\pi]$, transfer the given problem into an optimization problem in only one variable, and solve this problem using critical points.

(d) As in part (b), introduce new variables u and v so that the elliptic constraint is in standard form, eliminate either u or v to obtain two optimization problems in a single variable, and solve these two problems by the critical point method.