This solves the differential equation.

```
In[180]:= Clear["Global`*"]
DSolve[(x^2 + y[x]^2)^(3/2) == A (x + y[x] y'[x]) / x, y[x], x]
```

$$\text{Out[181]=}\ \left\{\left\{y[x] \to -\frac{i\sqrt{-4A^2 + x^6 + 4x^4 C[1] + 4x^2 C[1]^2}}{x^2 + 2C[1]}\right\}, \left\{y[x] \to \frac{i\sqrt{-4A^2 + x^6 + 4x^4 C[1] + 4x^2 C[1]^2}}{x^2 + 2C[1]}\right\}\right\}$$

Determining the constant c assuming that y = 0 if x is the equatorial radius a.

$$\text{In[182]:=}\ y[x\_] := \frac{\sqrt{4A^2 - x^6 - 4x^4 c - 4x^2 c^2}}{x^2 + 2c};$$

```
Solve[y[a] == 0, c]
```

$$\text{Out[183]=}\ \left\{\left\{c \to \frac{-a^3 - 2A}{2a}\right\}, \left\{c \to \frac{-a^3 + 2A}{2a}\right\}\right\}$$

Taking the first solution for c because otherwise positive terms becoming negative.

```
Clear["Global`*"]
```

$$c = \frac{-a^3 - 2A}{2a};$$

$$\text{FullSimplify}\left[\frac{\sqrt{4A^2 - x^6 - 4x^4 c - 4x^2 c^2}}{x^2 + 2c},\ \text{Assumptions} \to \{A > 0, a > 0, \text{Element}[x, \text{Reals}]\}\right]$$

$$-\frac{\sqrt{-(a-x)(a+x)\left(a^4 x^2 - \left(-2A + a x^2\right)^2\right)}}{a^3 + 2A - a x^2}$$

Testing the solution against known equatorial and polar radius ae and ap
assuming that b = y (0) yields the polar radius.

```
In[190]:= Clear["Global`*"]
g = 6.67 × 10^-11; T = 3600 × 24; w = 2 Pi / T; m = 5.976 × 10^24; A = g m / w^2;
V = 1.0832 × 10^21; r = (3 V / (4 Pi))^(1/3); ae = 6.3782 × 10^6; ap = 6.3568 × 10^6;
```

$$y[x\_, a\_] := \frac{\sqrt{-(a-x)(a+x)\left(a^4 x^2 - \left(-2A + a x^2\right)^2\right)}}{a^3 + 2A - a x^2};\ b = y[0, ae];$$

```
ae - ap
ae - b
Out[194]= 21400.
Out[195]= 10960.
```

In[157]:=
```
rho = m / V; f = 15 Pi / (4 g T^2 rho);
r (1 + f / 3)
r (1 - 2 f / 3)
```

Out[158]= 6.16772×10^7

Out[159]= 5.13416×10^7

Calculating the volume yields an equatorial radius a for any rotation period T, for instance T = 3 hours.

In[224]:=
```
Clear["Global`*"]
```

$$y[x\_] := \frac{\sqrt{-(a-x)\,(a+x)\,\left(a^4 x^2 - \left(-2 A + a x^2\right)^2\right)}}{a^3 + 2 A - a x^2};$$

```
Integrate[y[x]^2, {x, -a, a}, Assumptions → {a > 0, A > 0, Element[x, Reals]}]
```

Out[226]=
$$\frac{2 a^2 \left(-a^4 + a A + \frac{6 A^2 \operatorname{ArcTanh}\left[\sqrt{\frac{a^3}{a^3+2 A}}\right]}{\sqrt{a^4+2 a A}}\right)}{3 \left(a^3 + 2 A\right)}$$

In[255]:=
```
g = 6.67 × 10^-11; T = 3600 × 3; w = 2 Pi / T; m = 5.976 × 10^24; A = g m / w^2;
V = 1.0832 × 10^21; r = (3 V / (4 Pi)) ^ (1 / 3);
```

$$\text{sol} = \text{FindRoot}\left[V == \text{Pi}\,\frac{2 a^2 \left(-a^4 + a A + \frac{6 A^2 \operatorname{ArcTanh}\left[\sqrt{\frac{a^3}{a^3+2 A}}\right]}{\sqrt{a^4+2 a A}}\right)}{3 \left(a^3 + 2 A\right)}, \{a, r\}\right];$$

```
a = a /. sol
b = y[0]
a - b
```

Out[258]= 7.00674×10^6

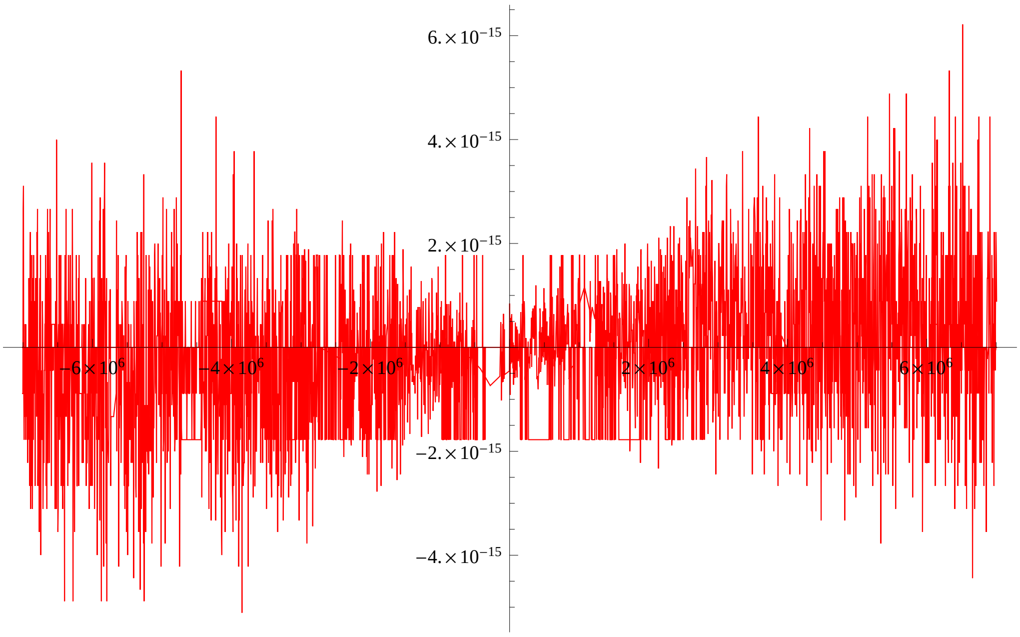
Out[259]= 6.11383×10^6

Out[260]= 892 908.

This is a test if the equilibrium of all forces is zero for all - a < x < a. As can be seen, this is the case.

In[261]:=
```
R[x_] := Sqrt[x^2 + y[x]^2]; S[x_] := -g m y[x] / R[x]^3 {y'[x], -1};
G[x_] := -g m / R[x]^3 {x, y[x]}; Z[x_] := w^2 {x, 0};
Plot[{S[x] + G[x] + Z[x]}, {x, -a, a}, PlotStyle → Red, PlotRange → All]
```

Out[262]=



Finally a plot of the curve (blue) for T = 3 days, the red curve is an ellipse (is slightly different).

In[263]:=
```
Curve = Plot[{y[x], -y[x]}, {x, -a, a}, PlotStyle → Blue];
Ellipse = ParametricPlot[{a Cos[p], b Sin[p]}, {p, 0, 2 Pi}, PlotStyle → Red];
Show[Curve, Ellipse, AspectRatio → Automatic]
```

Out[265]=