

Let's focus on the “middle” portion of Eq. 3.82,

$$(1) \quad 2\epsilon_{\alpha\gamma}\bar{u}_{1L\alpha}\bar{u}_{3L\gamma}\underbrace{\epsilon_{\beta\delta}(\sigma^\nu\bar{\sigma}^\lambda u_{2L})_\beta}_{\epsilon_{\beta\delta}(\sigma^\nu\bar{\sigma}^\lambda u_{2L})_\beta}(\sigma_\nu\bar{\sigma}_\lambda u_{4L})_\delta.$$

Expand in the spinor matrices,

$$\begin{aligned} & \epsilon_{\beta\delta}(\sigma^\nu\bar{\sigma}^\lambda u_{2L})_\beta \\ = & \epsilon_{\beta\delta}(\sigma^\nu)_{\beta m}(\bar{\sigma}^\lambda)_{mn}(u_{2L})_n \\ = & -\epsilon_{\delta\beta}(\sigma^\nu)_{\beta m}(\bar{\sigma}^\lambda)_{mn}(u_{2L})_n \quad \text{because } \epsilon_{\delta\beta} = -\epsilon_{\beta\delta} \\ = & -\underbrace{\epsilon_{\delta\beta}(\sigma^\nu)_{\beta m}}_{(\bar{\sigma}^\nu)_{\delta\beta}}(\bar{\sigma}^\lambda)_{mn}(u_{2L})_n \\ = & -(\bar{\sigma}^\nu)_{\delta\beta}\epsilon_{\beta m}(\bar{\sigma}^\lambda)_{mn}(u_{2L})_n \\ = & +(\bar{\sigma}^\nu)_{\beta\delta}(\bar{\sigma}^{\lambda T})_{nm}\epsilon_{m\beta}(u_{2L})_n \quad \text{because } (\bar{\sigma}^\lambda)_{mn} = (\bar{\sigma}^{\lambda T})_{nm} \\ = & (\bar{\sigma}^\nu)_{\beta\delta}\underbrace{(\bar{\sigma}^{\lambda T})_{nm}\epsilon_{m\beta}}_{\epsilon_{nm}(\sigma^\lambda)_{m\beta}}(u_{2L})_n \\ = & (\bar{\sigma}^\nu)_{\beta\delta}\epsilon_{nm}(\sigma^\lambda)_{m\beta}(u_{2L})_n \\ = & \epsilon_{nm}(u_{2L})_n(\sigma^\lambda\bar{\sigma}^\nu)_{m\delta}. \end{aligned}$$

Substituting this back into Eq. 1

$$\begin{aligned} & 2\epsilon_{\alpha\gamma}\bar{u}_{1L\alpha}\bar{u}_{3L\gamma}\underbrace{\epsilon_{\beta\delta}(\sigma^\nu\bar{\sigma}^\lambda u_{2L})_\beta}_{\epsilon_{\beta\delta}(\sigma^\nu\bar{\sigma}^\lambda u_{2L})_\beta}(\sigma_\nu\bar{\sigma}_\lambda u_{4L})_\delta \\ = & 2\epsilon_{\alpha\gamma}\bar{u}_{1L\alpha}\bar{u}_{3L\gamma}\epsilon_{nm}(u_{2L})_n(\sigma^\lambda\bar{\sigma}^\nu)_{m\delta}(\sigma_\nu\bar{\sigma}_\lambda u_{4L})_\delta \\ = & 2\epsilon_{\alpha\gamma}\bar{u}_{1L\alpha}\bar{u}_{3L\gamma}\epsilon_{nm}(u_{2L})_n\underbrace{(\sigma^\lambda\bar{\sigma}^\nu)_{m\delta}(\sigma_\nu\bar{\sigma}_\lambda u_{4L})_\delta}_{(\sigma^\lambda\bar{\sigma}^\nu\sigma_\nu\bar{\sigma}_\lambda u_{4L})_m} \\ = & 2\epsilon_{\alpha\gamma}\bar{u}_{1L\alpha}\bar{u}_{3L\gamma}\epsilon_{nm}(u_{2L})_n(\sigma^\lambda\bar{\sigma}^\nu\sigma_\nu\bar{\sigma}_\lambda u_{4L})_m. \end{aligned}$$

Substituting $m \rightarrow \delta$ and $n \rightarrow \beta$, we have the second line in Eq. 3.82.