First we determine our center of mass locations for the first arm with respect to the immovable pivot:

$$x_1 = \frac{l_1}{2}\sin\alpha \tag{1}$$

$$y_1 = -\frac{l_1}{2}\cos\alpha \tag{2}$$

We repeat for the second arm:

$$x_2 = l_1\sin\alpha + \frac{l_2}{2}\sin\beta \tag{3}$$

$$y_2 = -l_1\cos\alpha - \frac{l_2}{2}\cos\beta \tag{4}$$

We now solve for the first derivative (velocity) of the x and y coordinates of the CM for the first arm:

$$\dot{x}_1 = \frac{l_1}{2}\dot{\alpha}\cos\alpha \tag{5}$$

$$\dot{y}_1 = \frac{l_1}{2}\dot{\alpha}\sin\alpha \tag{6}$$

Taking these two equations, we can now solve for the magnitude of the velocity vector of the first arm's CM:

$$|v_1|^2 = \dot{x}_1^2 + \dot{y}_1^2 \tag{7}$$

$$|v_1|^2 = (\frac{\dot{\alpha}l_1}{2})^2 \tag{8}$$

We now solve for the velocity of the x and y coordinates of the CM for the second arm:

$$\dot{x}_2 = l_1\dot{\alpha}\cos\alpha + \frac{l_2}{2}\dot{\beta}\cos\beta \tag{9}$$

$$\dot{y}_2 = l_1\dot{\alpha}\sin\alpha + \frac{l_2}{2}\dot{\beta}\sin\beta \tag{10}$$

We can now combine these to find the magnitude of the velocity vector of the second arm's CM:

$$|v_2|^2 = \dot{x}_2^2 + \dot{y}_2^2 \tag{11}$$

$$(\dot{x_2})^2 = (l_1\dot{\alpha}\cos\alpha)^2 + (\frac{l_2}{2}\dot{\beta}\cos\beta)^2 + l_1l_2\dot{\alpha}\dot{\beta}\cos\alpha\cos\beta \tag{12}$$

$$(\dot{y_2})^2 = (l_1\dot{\alpha}\sin\alpha)^2 + (\frac{l_2}{2}\dot{\beta}\sin\beta)^2 + l_1l_2\dot{\alpha}\dot{\beta}\sin\alpha\sin\beta \tag{13}$$

$$|v_2|^2 = (l_1\dot{\alpha})^2 + (\frac{l_2}{2}\dot{\beta})^2 + l_1l_2\dot{\alpha}\dot{\beta}\cos(\alpha-\beta) \tag{14}$$

Having solved for the velocity squared of both CMs, we can derive the kinetic energy of each arm:

$$T_1 = \frac{1}{2} I_1 \dot{\alpha}^2 \tag{15}$$

$$T_2 = \frac{1}{2} I_2 \dot{\beta}^2 + \frac{1}{2} m_2 \left[(l_1 \dot{\alpha})^2 + (\frac{l_2}{2} \dot{\beta})^2 + l_1 l_2 \dot{\alpha} \dot{\beta} \cos(\alpha - \beta) \right] \tag{16}$$

After solving for velocities of the CM of both arms, we can now define the Lagrangian:

$$\mathcal{L} = T - V \tag{17}$$

Which is the difference in kinetic (T), and potential (V) energies. In this horizontally planar structure, the potential energy term is zero. The Lagrangian is therefore the sum of only the kinetic energy terms:

$$\mathcal{L} = \frac{1}{2} I_1 \dot{\alpha}^2 + \frac{1}{2} I_2 \dot{\beta}^2 + \frac{1}{2} m_2 \left[(l_1 \dot{\alpha})^2 + (\frac{l_2}{2} \dot{\beta})^2 + l_1 l_2 \dot{\alpha} \dot{\beta} \cos(\alpha - \beta) \right] \tag{18}$$

$$\mathcal{L} = \frac{I_1 \dot{\alpha}^2}{2} + \frac{I_2 \dot{\beta}^2}{2} + \frac{m_2 l_1^2 \dot{\alpha}^2}{2} + \frac{m_2 l_2^2 \dot{\beta}^2}{8} + \frac{m_2 l_1 l_2 \dot{\alpha} \dot{\beta} \cos(\alpha - \beta)}{2} \tag{19}$$

The next step is to actually solve for the angular accelerations about each pivot using the following equations:

$$\frac{\partial}{\partial t} \left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i} \right) - \frac{\partial \mathcal{L}}{\partial q_i} = \Sigma M_i \tag{20}$$

Where q_i is α and β, and the right hand side is the sum of moments about the joint in question. Solving for the various partial derivatives and moments:

$$\frac{\partial \mathcal{L}}{\partial \alpha} = \frac{-m_2 l_1 l_2 \dot{\alpha} \dot{\beta} \sin(\alpha - \beta)}{2} \tag{21}$$

$$\frac{\partial \mathcal{L}}{\partial \beta} = \frac{m_2 l_1 l_2 \dot{\alpha} \dot{\beta} \sin(\alpha - \beta)}{2} \tag{22}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\alpha}} = I_1 \dot{\alpha} + m_2 l_1^2 \dot{\alpha} + \frac{m_2 l_1 l_2 \dot{\beta} \cos(\alpha - \beta)}{2} \tag{23}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\beta}} = I_2 \dot{\beta} + \frac{m_2 l_2^2 \dot{\beta}}{4} + \frac{m_2 l_1 l_2 \dot{\alpha} \cos(\alpha - \beta)}{2} \tag{24}$$

$$\frac{\partial}{\partial t} \left(\frac{\partial \mathcal{L}}{\partial \dot{\alpha}} \right) = I_1 \ddot{\alpha} + m_2 l_1^2 \ddot{\alpha} + \frac{m_2 l_1 l_2 \ddot{\beta} \cos(\alpha - \beta)}{2} - \frac{m_2 l_1 l_2 \dot{\beta} \sin(\alpha - \beta)(\dot{\alpha} - \dot{\beta})}{2} \tag{25}$$

$$\frac{\partial}{\partial t} \left(\frac{\partial \mathcal{L}}{\partial \dot{\beta}} \right) = I_2 \ddot{\beta} + \frac{m_2 l_2^2 \ddot{\beta}}{4} + \frac{m_2 l_1 l_2 \ddot{\alpha} \cos(\alpha - \beta)}{2} - \frac{m_2 l_1 l_2 \dot{\alpha} \sin(\alpha - \beta)(\dot{\alpha} - \dot{\beta})}{2} \tag{26}$$

$$\Sigma M_\alpha = l_v F_1 - \mu_1 \dot{\alpha} \tag{27}$$

$$\Sigma M_\beta = l_v F_2 - \mu_2 \dot{\beta} \tag{28}$$

Solve for α by plugging equations (21), (25), and (27) into Lagrange's equation (20):

$$I_1\ddot{\alpha} + m_2 l_1^2 \ddot{\alpha} + \frac{m_2 l_1 l_2 \ddot{\beta}\cos(\alpha-\beta)}{2} + \frac{m_2 l_1 l_2 \dot{\beta}^2 \sin(\alpha-\beta)}{2} = l_v F_1 - \mu_1 \dot{\alpha} \tag{29}$$

Solve for β by plugging equations (22), (26), and (28) into Lagrange's equation (20):

$$I_2\ddot{\beta} + \frac{m_2 l_2^2 \ddot{\beta}}{4} + \frac{m_2 l_1 l_2 \ddot{\alpha}\cos(\alpha-\beta)}{2} - \frac{m_2 l_1 l_2 \dot{\alpha}^2 \sin(\alpha-\beta)}{2} = l_v F_2 - \mu_2 \dot{\beta} \tag{30}$$