A process can be represented by the first order equation

$$\tau \frac{\mathrm{d}y(t)}{\mathrm{d}t} + y(t) = Ku(t)$$

Determine the transfer function of the process.

If the input is a step change in $u(t)$, determine the value of $y(t)$ when $K = 6$, $t = 15$ s and $\tau = 10$ s.

Rearranging the equation

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} + \frac{1}{\tau}y(t) = \frac{Ku(t)}{\tau}$$

Using the Laplace transforms given in the earlier table

$$\left[sY(s) + y(-0)\right] + \frac{Y(s)}{\tau} = \frac{KU(s)}{\tau}$$

Assuming the initial condition was steady state ($y(-0) = 0$) we get

$$sY(s) + \frac{Y(s)}{\tau} = \frac{KU(s)}{\tau}$$

or $$Y(s)\left(s + \frac{1}{\tau}\right) = \frac{KU(s)}{\tau}$$

The transfer function in the s domain, $G(s)$, is input, $Y(s)$, over output, $U(s)$, so we can rearrange the above equation to give

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\frac{K}{\tau}}{\left(s + \frac{1}{\tau}\right)} = \frac{K}{\tau\left(s + \frac{1}{\tau}\right)}$$

$$= \frac{K}{\tau s + 1}$$

Having obtained the transfer function, you are now asked to determine the value of $y(t)$.

To do this, we need to perform the inverse transform of $Y(s)$ to return to the time domain.

We therefore need to re-arrange the transfer function equation for when the input change is a step change i.e.

$$U(s) = \frac{1}{s}$$

$$Y(s) = \frac{\frac{K}{\tau}}{\left(s + \frac{1}{\tau}\right)} \times \frac{1}{s} = \frac{\frac{K}{\tau}}{s\left(s + \frac{1}{\tau}\right)}$$

This is now in the form of $\frac{a}{s(s + a)}$, where $a = \frac{1}{\tau}$.

The inverse transform of this is $1 - e^{-at}$ (from the table)

So
$$y(t) = K\left(1 - e^{-at}\right) = K\left(1 - e^{-\frac{t}{\tau}}\right)$$

Note, that you may have arrived at the same answer using a different route!

When $K = 6, \tau = 10\text{ s}$ and $t = 15\text{ s}$

Then
$$y(15) = 6\left(1 - e^{-\frac{15}{10}}\right)$$

$$= 6 \times 0.777$$

$$= 4.662 \text{ units}$$