The University of Nottingham

SCHOOL OF PHYSICS & ASTRONOMY

A LEVEL 1 MODULE, AUTUMN SEMESTER 2006-2007

**ANALYTICAL SKILLS AND MODELLING/
MATHEMATICAL AND COMPUTATIONAL MODELLING**

Time allowed ONE hour and THIRTY minutes

Candidates must NOT start writing their answers until told to do so

Answer Section A and Section B

NO CALCULATORS ARE PERMITTED IN THIS EXAMINATION

Dictionaries are not allowed with one exception. Those whose first language is not English may use a dictionary to translate between that language and English provided that neither language is the subject of this examination.

No electronic devices capable of storing and retrieving text, including electronic dictionaries may be used.

*An indication is given of the approximate weighting of each part of a question by means of a bold figure enclosed by curly brackets, e.g. {**2**}, immediately following that part.*

DO NOT turn examination paper over until instructed to do so

Speed of light in free space	c	3.00×10^{8} m s^{-1}
Gravitational constant	G	6.67×10^{-11} N m^{2} kg^{-2}
Planck's constant	h	6.63×10^{-34} J s
	$\hbar$	1.055×10^{-34} J s
Elementary charge	e	1.60×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	1.6726×10^{-27} kg
Mass of neutron	m_n	1.6749×10^{-27} kg
Boltzmann's constant	k_B	1.38×10^{-23} J K^{-1}
Gas constant	R	8.31 J K^{-1} mol^{-1}
Permittivity of free space	ε_0	8.85×10^{-12} F m^{-1}
Permeability of free space	μ_0	$4\pi\times10^{-7}$ H m^{-1}
Bohr magneton	μ_B	9.27×10^{-24} J T^{-1}
Stefan-Boltzmann constant	σ	5.567×10^{-8} W m^{-2} K^{-4}
Avogadro's number	N_A	6.02×10^{26} kmol^{-1}

SECTION A

You may answer as many questions as you wish. Maximum credit of 10 marks will be awarded and any marks in excess of 10 will **not** be counted. You should aim to spend about 30 minutes on this section.

1. Poiseuille's equation, for the flow of a viscous fluid along a circular tube of radius R and length L, states that the volume flow per second Q is

$$Q = \frac{\Delta p}{8L\eta}\pi R^4 ,$$

where Δp is the pressure difference between the ends of the tube. The coefficient of viscosity η is given by

$$\eta = \frac{1}{3}m\lambda n\bar{v} ,$$

where m, λ and n are all constants, and $\bar{v}$, the average microscopic molecular speed, is proportional to the square root of the temperature. If the circular tube is made from a material with coefficient of linear expansion α, derive an expression for the fractional change in Q in terms of T, ΔT and α when the temperature is increased by a small amount from T to $T + \Delta T$. **{5}**

[You may assume that $\alpha = \frac{1}{l}\frac{\mathrm{d}l}{\mathrm{d}T}$ and that Δp is constant.]

2. A particle of mass m moves in one dimension, under the influence of a force, with potential:

$$V(x) = \frac{x^2}{1+x^2} ,$$

where x is the distance from the origin of co-ordinates. Derive an expression for the force acting on the particle and calculate the value of x at equilibrium. **{2}** Sketch the form of the potential over the range $-5 < x < 5$, giving brief details of how you decided on the shape of your graph. **{2}** If the particle has velocity u at $x = 0$, what is the maximum value of u such that the particle will eventually return to the equilibrium position? **{1}**

3. In the Hall effect, a piece of current carrying semiconductor will generate a voltage across two of its faces when a magnetic field is applied perpendicular to the direction of current flow. This so-called Hall voltage (V_H) is related to the size of the applied field (B), the current (I) flowing in the semiconductor, the amount of free electronic charge per unit volume (λ) and the linear dimension of the sample (z).

(a) Write down the dimensions of the parameter λ. **{1}**

(b) Assuming that the dimensions of B and V_H are $[MQ^{-1}T^{-1}]$ and $[ML^2T^{-2}Q^{-1}]$ respectively, where [Q] represents the dimension of charge, use dimensional analysis to derive a possible expression for V_H in terms of B, I, λ, and z. **{3}**

(c) Hence calculate the number of electrons per unit volume in a 1 cm thick piece of semiconductor carrying a current of 1 A, if the magnitude of the applied B field is 48×10^{-3} Tesla and the measured Hall voltage is 1×10^{-6} V. **{1}**

SECTION B

Answer ONE question only. Maximum credit of 20 marks will be awarded. You should aim to spend about 60 minutes on this section.

4. An object of mass m falls from rest under the action of gravity and experiences a drag force that is proportional to its speed v.

(a) Sketch a diagram of the forces acting on the particle and hence show that the differential equation describing the motion of the particle is given by

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = mg - kv,$$

where k is a positive constant. **{2}**

(b) Solve this equation and show that the general solution for v as a function of time takes the form

$$v = A(1 - exp(-Bt))$$

and derive expressions for A and B. **{7}**

(c) Derive an expression for the limiting value of the velocity v as t tends to infinity (i.e. the terminal velocity) using

(i) the limiting behaviour of v as a function of t,

(ii) directly from the differential equation. **{2}**

(d) Sketch a graph of the variation of the velocity of the particle as a function of time. **{2}**

In the Millikan oil drop experiment, tiny charged drops fall through the air under the action of gravity. Measurements of the velocity of the drops are made when they have almost reached their terminal velocity.

(e) Find an expression for the time taken for a drop to reach 99% of its terminal velocity. **{5}**

(f) Calculate this time if the terminal velocity is 0.1 cm s^{-1}. (Assume that g = 10 m s^{-2} and that $\log_e 10 = 2.3$.) **{2}**

5. For certain weakly-bound solids, the potential between atoms can be written as a function of R, the atomic spacing, as

$$V(R) = A\left[\left(\frac{B}{R}\right)^{12} - \left(\frac{B}{R}\right)^{6}\right],$$

where A and B are constants. Show that the equilibrium separation of atoms which occurs at the minimum value of $V(R)$ is given by

$$R_0 = B(2)^{1/6}. \quad \{4\}$$

By considering small displacements, x, from R_0 (i.e. $R = R_0 + x$ where $x \ll R_0$) and substituting in $V(R)$, show that the potential may be expressed in the form

$$V = P + Qx^2$$

and find expressions for P and Q in terms of A and B. **{10}**

Show that the same result may be obtained by expanding the potential as an harmonic series about the equilibrium position, R_0 in the form:

$$V(R) = V(R_0) + \frac{x^2}{2}\left(\frac{d^2V}{dR^2}\right)_{R=R_0} + \quad \ldots\ldots\ldots\ldots \qquad \{6\}$$

End