The University of Tennessee at Martin

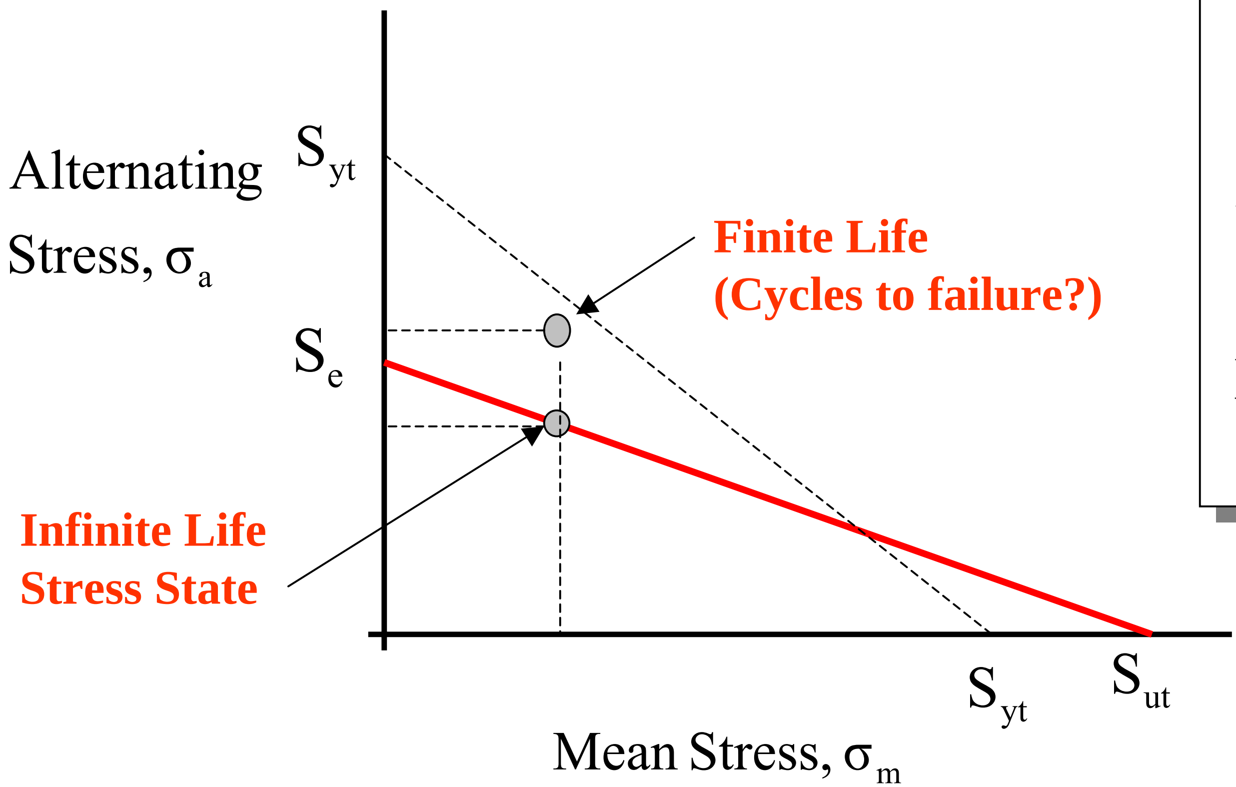
MARTIN School of Engineering

Fatigue II

Lecture 11

Engineering 473
Machine Design

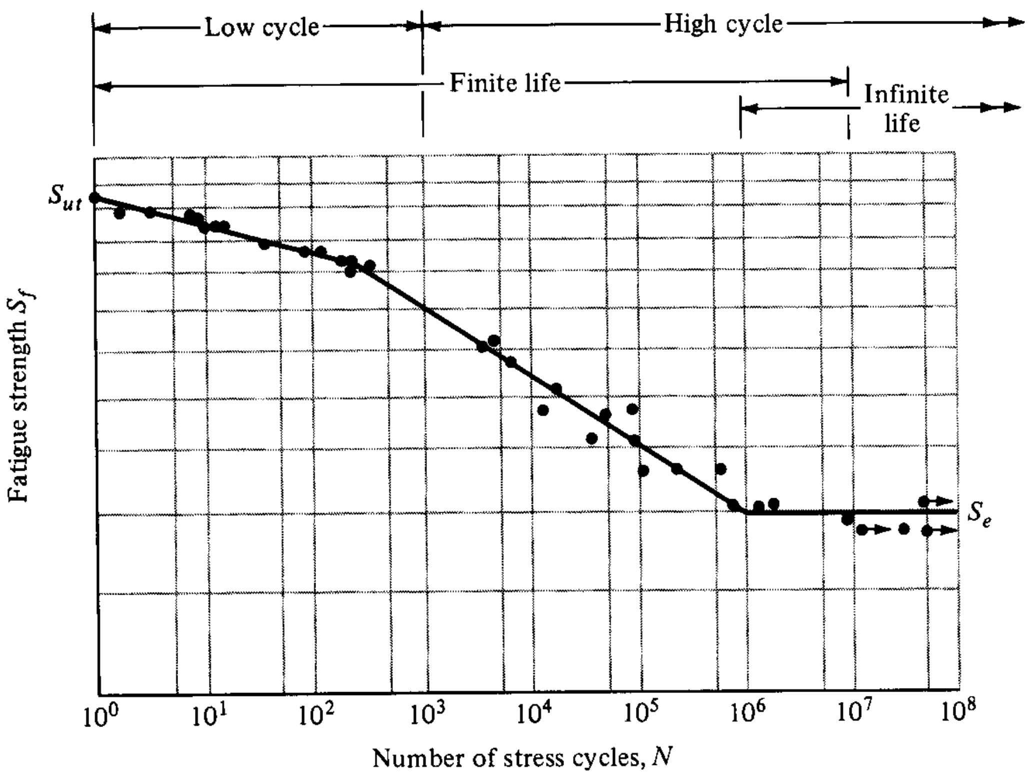
Finite Life Estimates



Goodman Diagram

How can the life of a part be estimated if the mean stress-alternating stress pair lie above the Goodman line?

S-N Curve



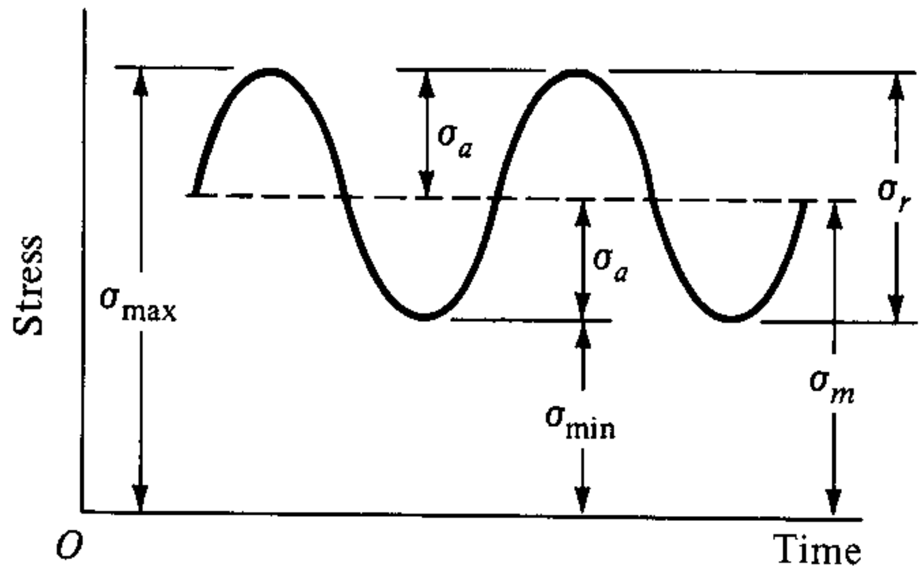
Completely reversed cyclic stress, UNS G41200 steel

The S-N curve gives the cycles to failure for a completely reversed (R=-1) uniaxial stress state.

What do you do if the stress state is not completely reversed?

Shigley, Fig. 7-6

Definitions



Stress Range

$$\sigma_r = \sigma_{max} - \sigma_{min}$$

Alternating Stress

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Mean Stress

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

Stress Ratio

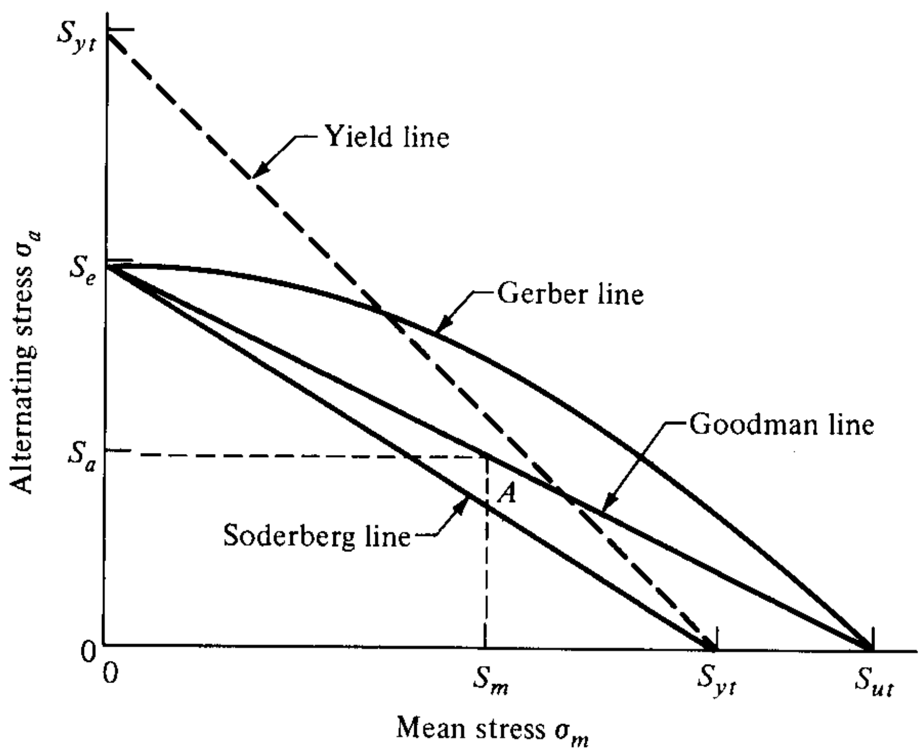
$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

Amplitude Ratio

$$A = \frac{\sigma_a}{\sigma_m}$$

Note that R=-1 for a completely reversed stress state with zero mean stress.

Fluctuating-Stress Failure Interaction Curves



The interaction curves provide relationships between alternating stress and mean stress.

When the mean stress is zero, the alternating component is equal to the endurance limit.

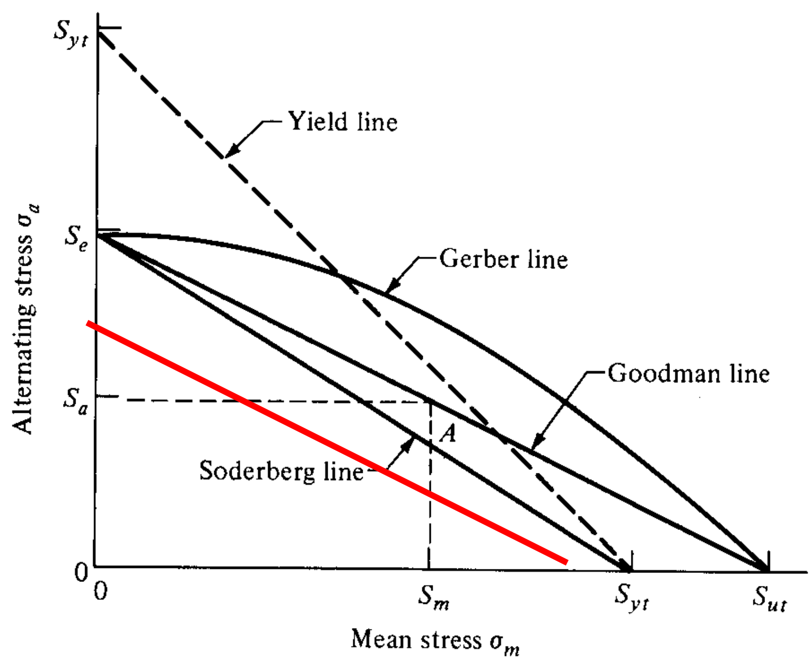
The interaction curves are for infinite life or a large number of cycles.

Shigley, Fig. 7-16

Goodman Interaction Line

$$\frac{k_f S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$$

Any combination of mean and alternating stress that lies on or below Goodman line will have infinite life.

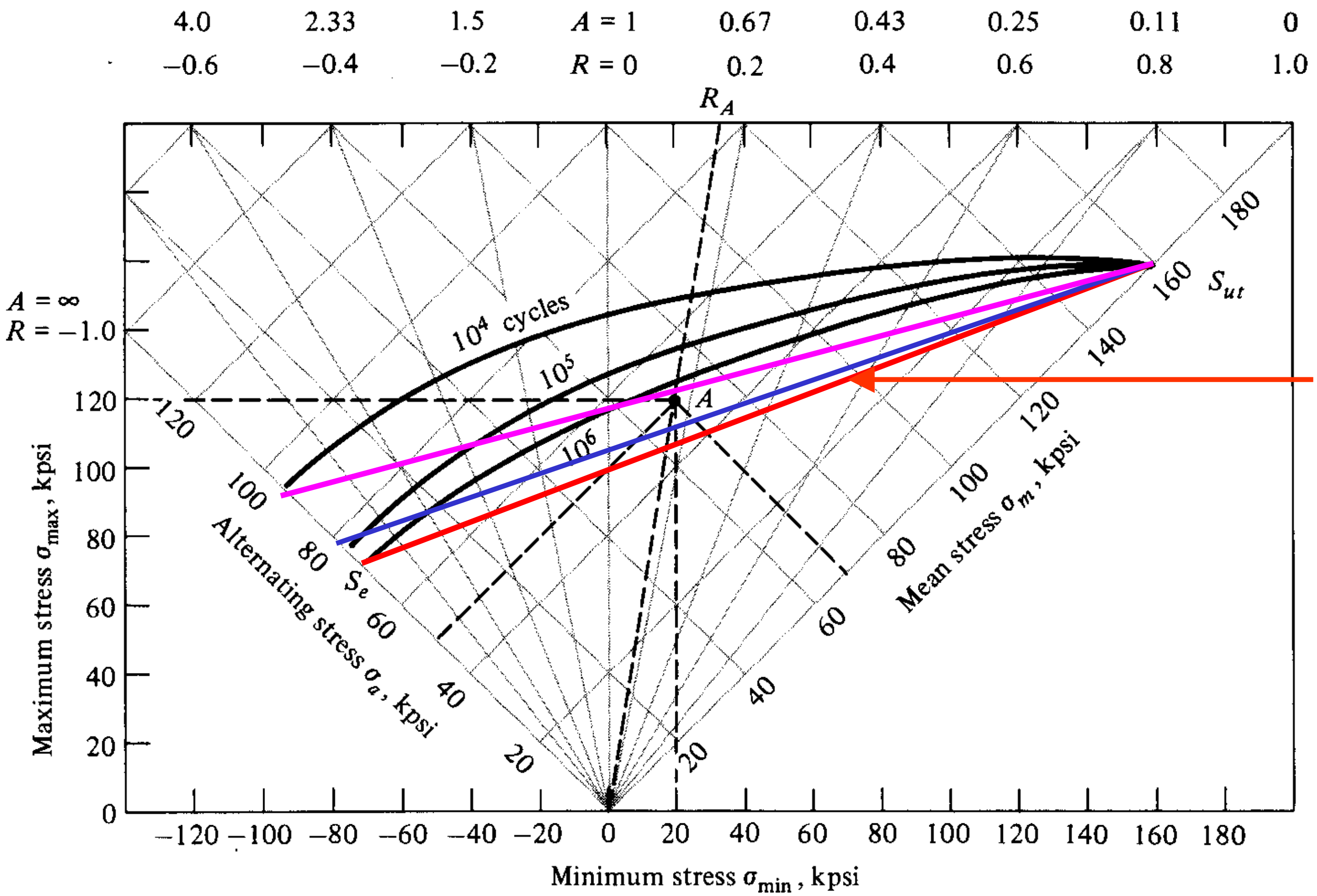


Factor of Safety Format

$$\frac{k_f S_a}{S_e} + \frac{S_m}{S_{ut}} = \frac{1}{N_f}$$

Note that the fatigue stress concentration factor is applied only to the alternating component.

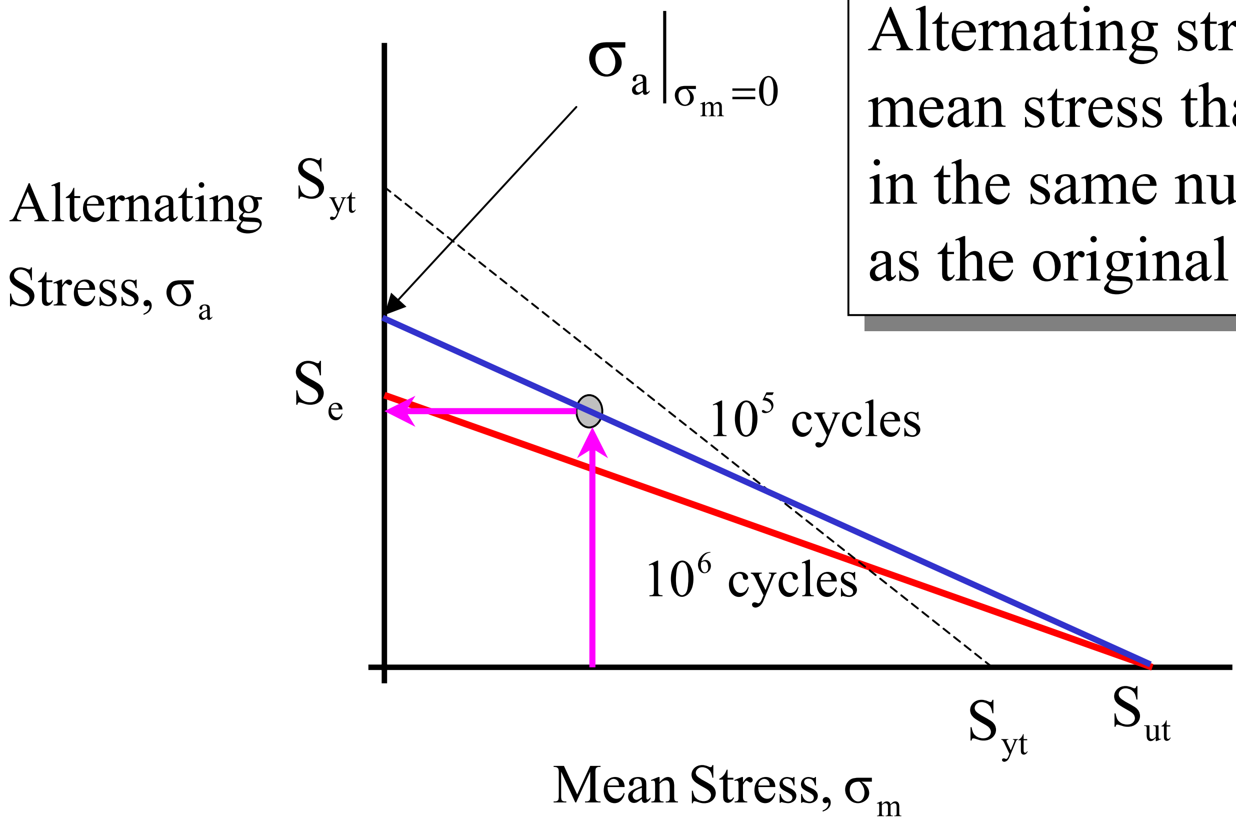
Master Fatigue Plot



Constant cycles till failure interaction curves.

Shigley, Fig. 7-15

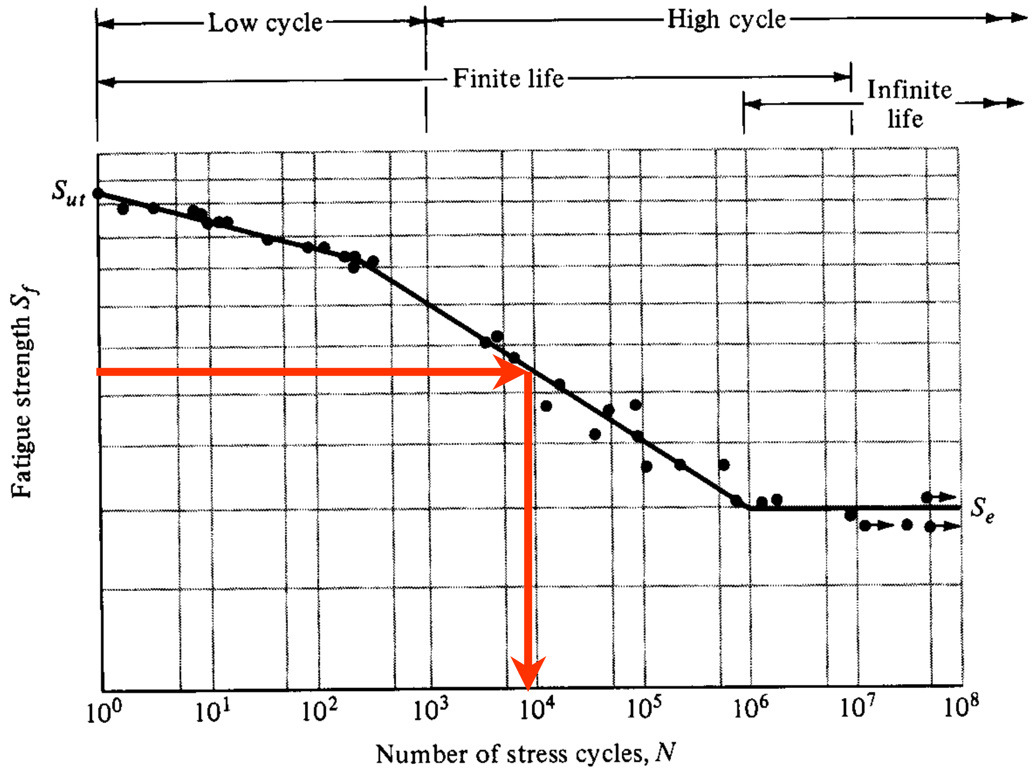
Equivalent Alternating Stress



Alternating stress at zero mean stress that fails the part in the same number of cycles as the original stress state.

The red and blue lines are estimated fatigue interaction curves associated with a specific number of cycles to failure.

Number of Cycles to Failure



Once the equivalent alternating stress is found, the S-N curve may be used to find the number of cycles to failure.

Equivalent Alternating Stress Formula

$$\frac{k_f \sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{N_f}$$

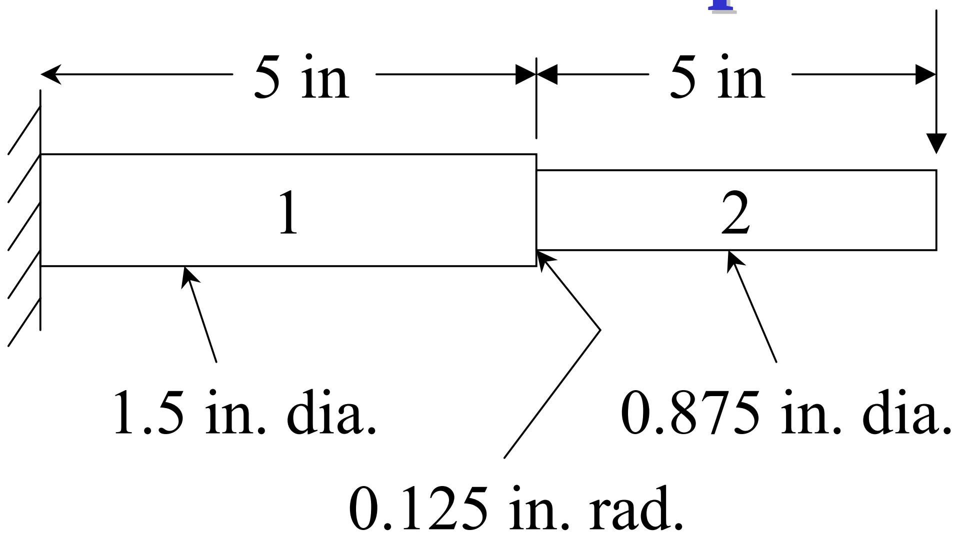
Goodman Line

$$\frac{k_f \sigma_a}{\sigma_a\big|_{\sigma_m=0}} + \frac{\sigma_m}{S_{ut}} = \frac{1}{N_f}$$

$$\sigma_a\big|_{\sigma_m=0} = \frac{k_f \sigma_a}{\dfrac{1}{N_f} - \dfrac{\sigma_m}{S_{ut}}}$$

$\sigma_a\big|_{\sigma_m=0} \equiv$ Equivalent completely reversed ($R = -1$) stress that causes fatigue failure in the same number of cycles as the original σ_a and σ_m pair.

Example



$$P_{max} = 3000\,\text{lb}$$

$$P_{min} = 2000\,\text{lb}$$

Material UNS G41200 Steel

Notch sensitivity q=0.3

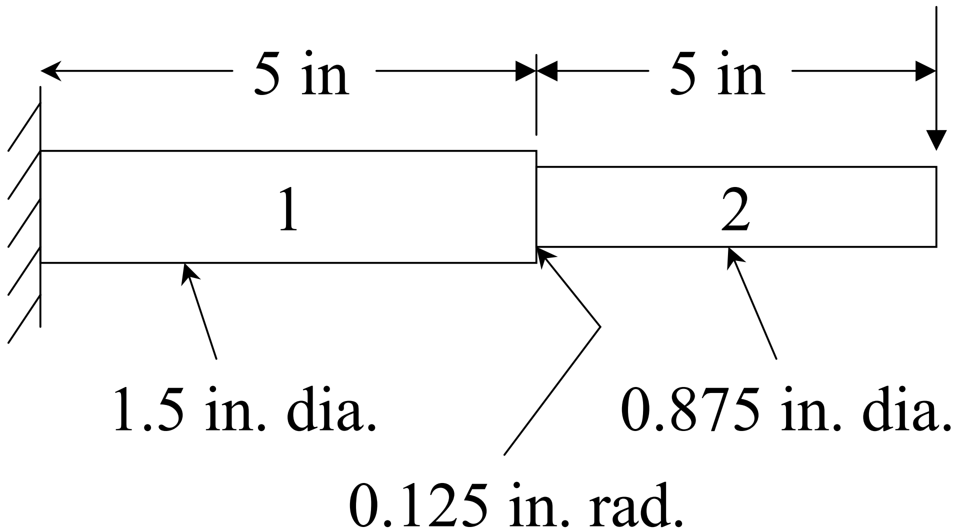
$$I_1 = \frac{\pi}{64} D_1^4 = \frac{\pi}{64}(1.5)^4 = 0.249\,\text{in}^4$$

$$I_2 = \frac{\pi}{64} D_2^4 = \frac{\pi}{64}(0.875)^4 = 0.088\,\text{in}^4$$

$$S_1 = \frac{I_1}{c_1} = \frac{0.249\,\text{in}^4}{0.75\,\text{in}} = 0.332\,\text{in}^3$$

$$S_2 = \frac{I_2}{c_2} = \frac{0.088\,\text{in}^4}{0.438\,\text{in}} = 0.201\,\text{in}^3$$

Example
(Continued)



$P_{max} = 3000\,\text{lb}$

$P_{min} = 2000\,\text{lb}$

Material UNS G41200 Steel

Notch sensitivity q=0.3

$$q = \frac{k_f - 1}{k_t - 1}$$

$$k_f = 1 + q(k_t - 1)$$

$$\frac{D}{d} = \frac{1.5\,\text{in}}{0.875\,\text{in}} = 1.71$$

$$\frac{r}{d} = \frac{0.125}{0.875} = 0.143$$

Ref. Peterson

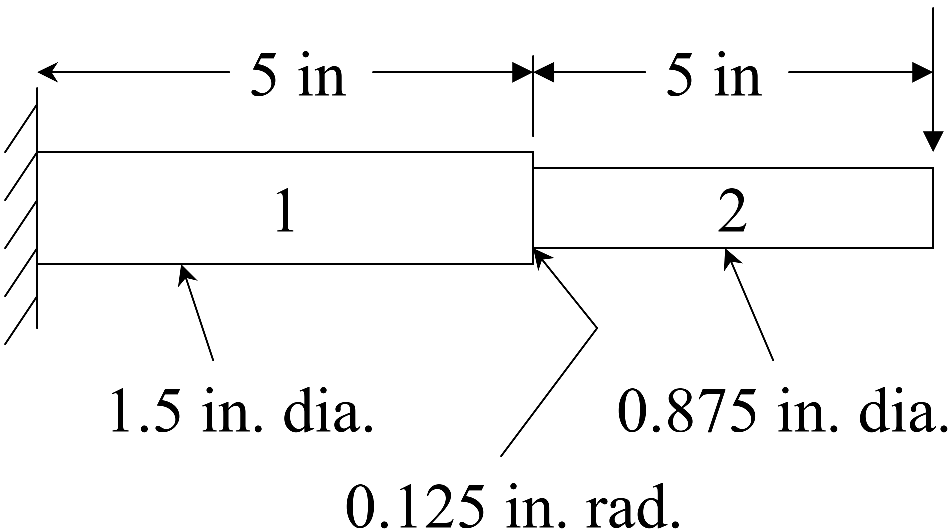
$$k_t = 1.61$$

$$k_f = 1 + q(k_t - 1)$$

$$= 1 + 0.3(1.61 - 1)$$

$$= 1.18$$

Example
(Continued)



$$P_{max} = 3000 \text{ lb}$$
$$P_{min} = 2000 \text{ lb}$$

Material UNS G41200 Steel

Notch sensitivity q=0.3

Section 1 (Base)

$$\sigma_{max} = \frac{M_1}{S_1} = \frac{(3000 \text{ lb})(10 \text{ in})}{0.332 \text{ in}^3} = 90.4 \text{ ksi}$$

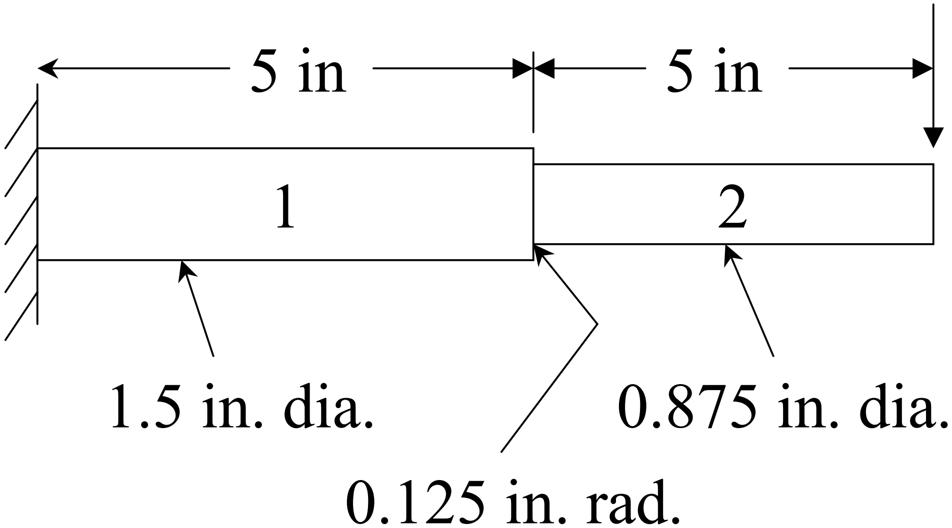
$$\sigma_{min} = \frac{M_1}{S_1} = \frac{(2000 \text{ lb})(10 \text{ in})}{0.332 \text{ in}^3} = 60.2 \text{ ksi}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = 15.1 \text{ ksi}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 75.3 \text{ ksi}$$

Example

(Continued)



$$P_{max} = 3000\,\text{lb}$$

$$P_{min} = 2000\,\text{lb}$$

Material UNS G41200 Steel

Notch sensitivity q=0.3

Section 2 (Fillet)

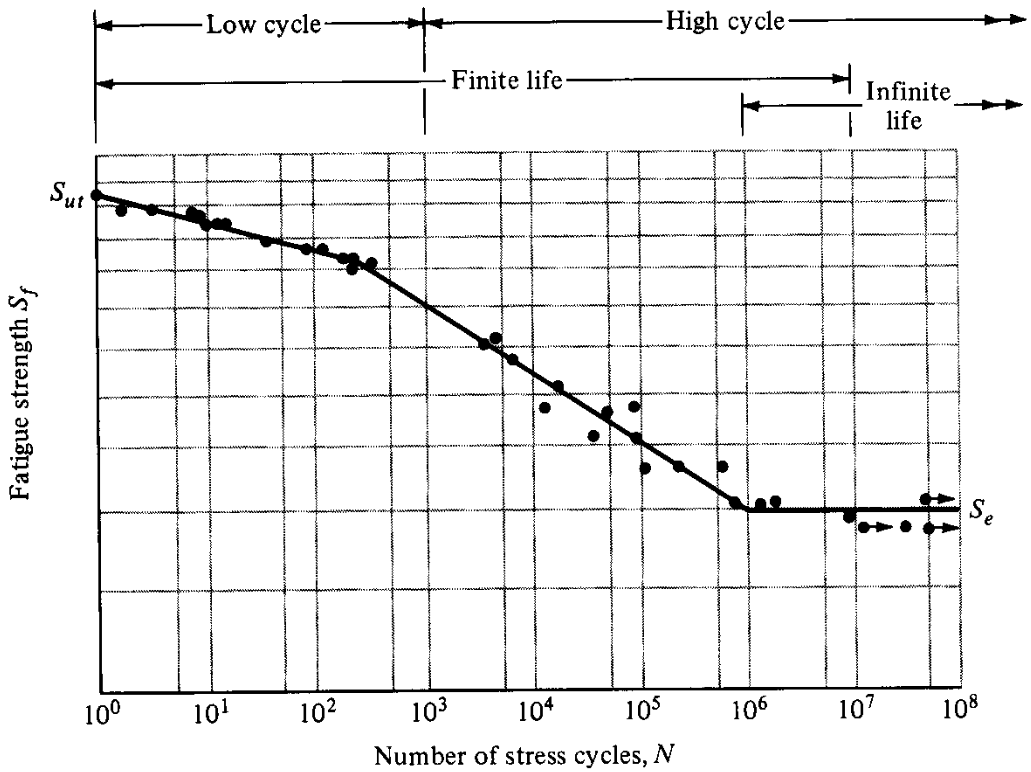
$$\sigma_{max} = \frac{M_1}{S_1} = \frac{(3000\,\text{lb})(5\,\text{in})}{0.201\,\text{in}^3} = 74.6\,\text{ksi}$$

$$\sigma_{min} = \frac{M_1}{S_1} = \frac{(2000\,\text{lb})(5\,\text{in})}{0.201\,\text{in}^3} = 49.8\,\text{ksi}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = 12.4\,\text{ksi}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 62.2\,\text{ksi}$$

Example
(Continued)



$$S_{ut} = 116\,\text{ksi}$$

$$S'_e = 30\,\text{ksi} = S_e$$

$$\frac{k_f \sigma_a}{S_e} + \frac{\sigma_m}{S_{ult}} = \frac{1}{N_f}$$

Completely reversed cyclic stress, UNS G41200 steel

Shigley, Fig. 7-6

Example
(Continued)

Section 1 (Base)

$$\sigma_{max} = \frac{M_1}{S_1} = \frac{(3000\text{ lb})(10\text{ in})}{0.332\text{ in}^3} = 90.4\text{ ksi}$$

$$\sigma_{min} = \frac{M_1}{S_1} = \frac{(2000\text{ lb})(10\text{ in})}{0.332\text{ in}^3} = 60.2\text{ ksi}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = 15.1\text{ ksi}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 75.3\text{ ksi}$$

$$S_{ut} = 116\text{ ksi}$$

$$S'_e = 30\text{ ksi} = S_e$$

$$\frac{k_f \sigma_a}{S_e} + \frac{\sigma_m}{S_{ult}} = \frac{1}{N_f}$$

$$N_f = 1$$

$$\frac{1.0(15.1\text{ ksi})}{30\text{ ksi}} + \frac{75.3\text{ ksi}}{116\text{ ksi}} = 1.15$$

Part has finite life at base.

Example
(Continued)

Section 2 (Fillet)

$$\sigma_{max} = \frac{M_1}{S_1} = \frac{(3000\ \mathrm{lb})(5\ \mathrm{in})}{0.201\ \mathrm{in}^3} = 74.6\ \mathrm{ksi}$$

$$\sigma_{min} = \frac{M_1}{S_1} = \frac{(2000\ \mathrm{lb})(5\ \mathrm{in})}{0.201\ \mathrm{in}^3} = 49.8\ \mathrm{ksi}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = 12.4\ \mathrm{ksi}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 62.2\ \mathrm{ksi}$$

$$S_{ut} = 116\ \mathrm{ksi}$$

$$S'_e = 30\ \mathrm{ksi} = S_e$$

$$\frac{k_f \sigma_a}{S_e} + \frac{\sigma_m}{S_{ult}} = \frac{1}{N_f}$$

$$N_f = 1$$

$$\frac{1.18(12.4\ \mathrm{ksi})}{30\ \mathrm{ksi}} + \frac{62.2\ \mathrm{ksi}}{116\ \mathrm{ksi}} = 1.02$$

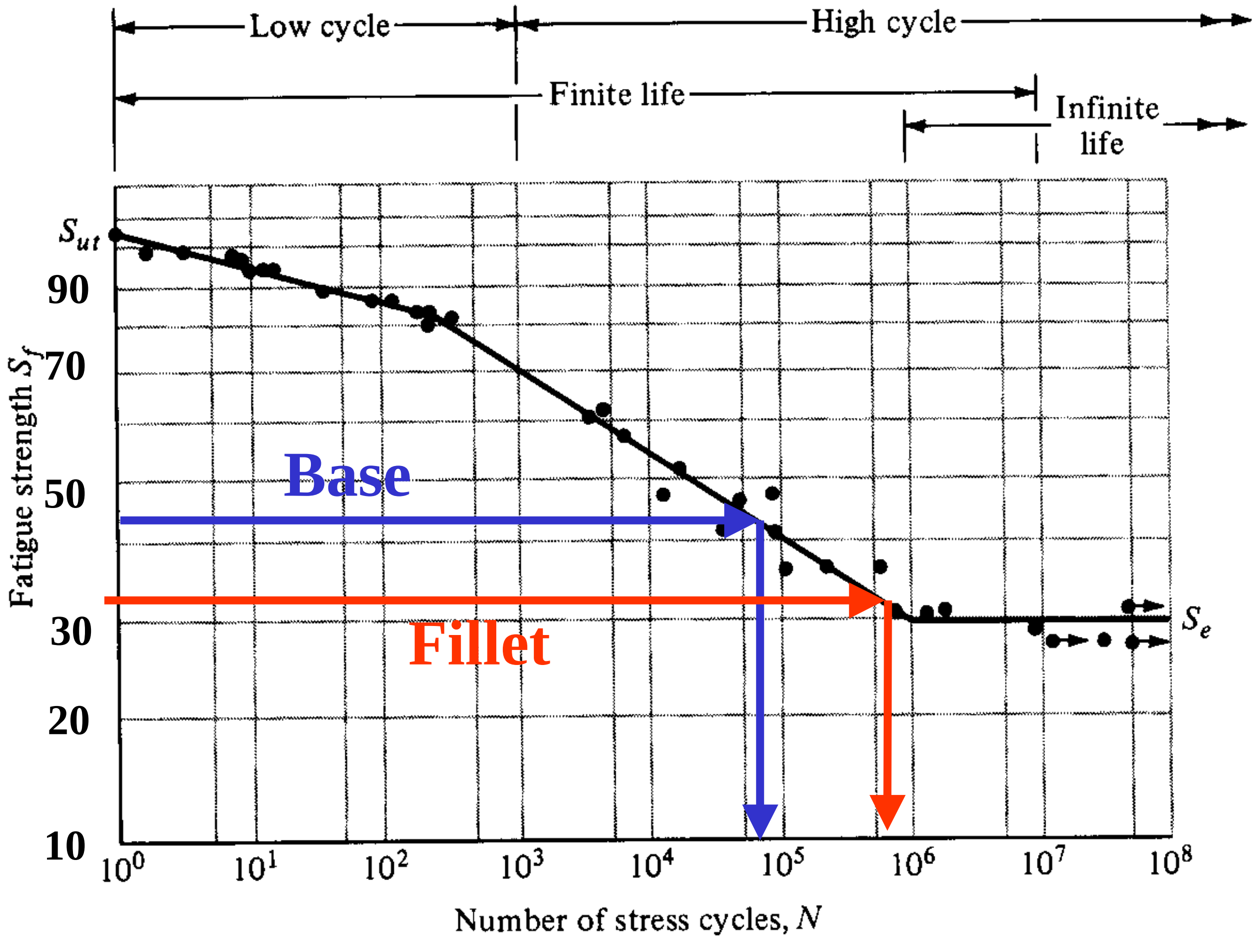
Part has finite life.

Calculation of Equivalent Alternating Stress

$$\sigma_a\big|_{\sigma_m=0} = \frac{k_f \sigma_a}{\dfrac{1}{N_f} - \dfrac{\sigma_m}{S_{ut}}}$$

Base	**Fillet**
$\sigma_a = 15.1\,\text{ksi}$	$\sigma_a = 12.4\,\text{ksi}$
$\sigma_m = 75.3\,\text{ksi}$	$\sigma_m = 62.2\,\text{ksi}$
$\sigma_a\big\|_{\sigma_m=0} = \dfrac{(1.0)15.1}{\dfrac{1}{1.0} - \dfrac{75.3}{116}}$	$\sigma_a\big\|_{\sigma_m=0} = \dfrac{(1.18)12.4}{\dfrac{1}{1.0} - \dfrac{62.2}{116}}$
$= 43.0\,\text{ksi}$	$= 31.5\,\text{ksi}$

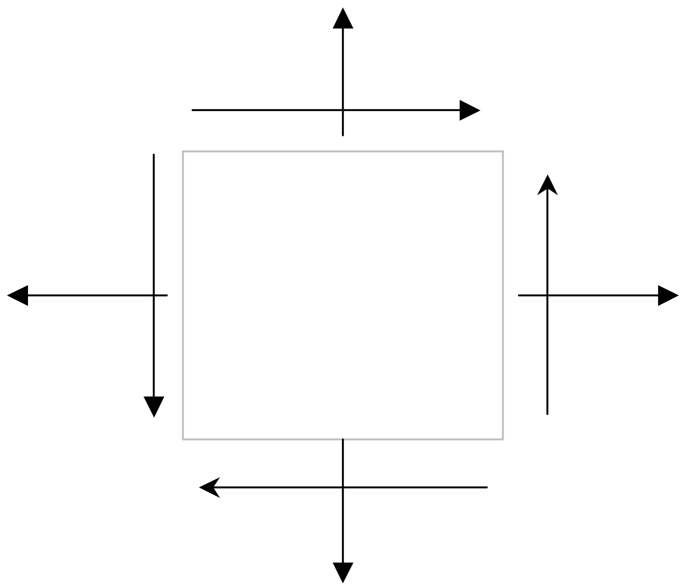
Cycles to Failure Estimate



Multi-axis Fluctuating Stress States

Everything presented on fatigue has been based on experiments involving a single stress component.

What do you do for problems in which there are more than one stress component?



Marin Load Factor, k_c

$$S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot S_e'$$

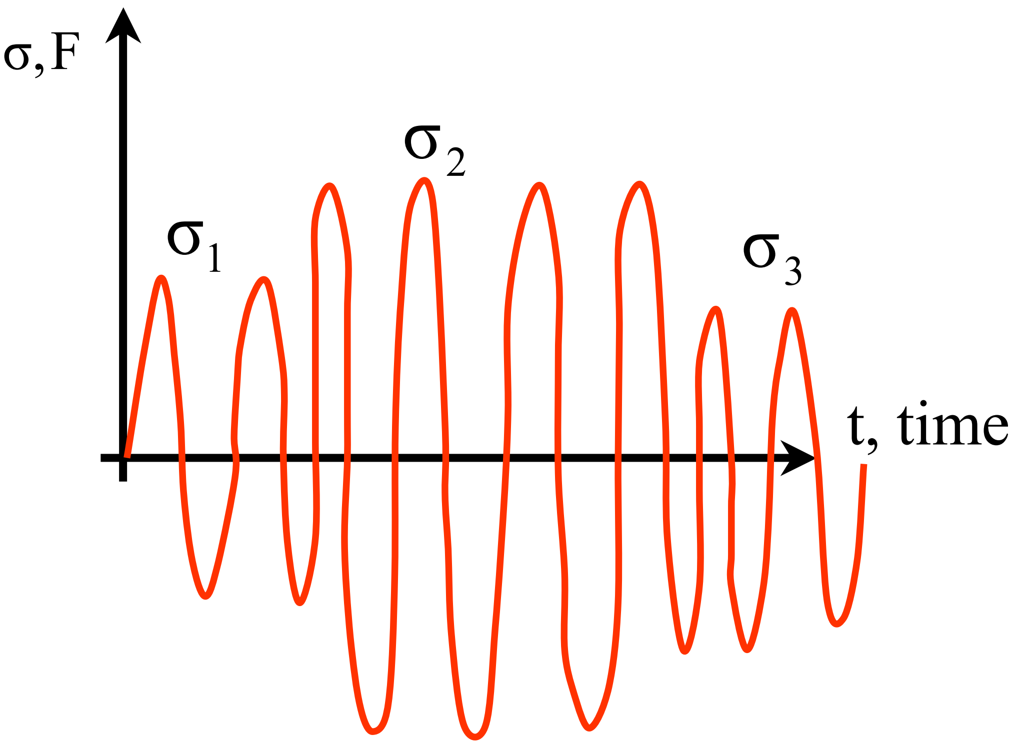
The endurance limit is a function of the load/stress component used in the test.

$$k_c = \begin{cases} 0.923 & \text{Axial loading} \quad S_{ut} \le 220\ \text{ksi}\ (1520\ \text{MPa}) \\ 1 & \text{Axial loading} \quad S_{ut} > 220\ \text{ksi}\ (1520\ \text{MPa}) \\ 1 & \text{Bending} \\ 0.577 & \text{Torsion and shear} \end{cases}$$

Alternating and Mean Von Mises Stresses

1. Increase the stress caused by an axial force by $1/k_{c}$.
2. Multiply each stress component by the appropriate fatigue stress concentration factor.
3. Compute the maximum and minimum von Mises stresses.
4. Compute the alternating and mean stresses based on the maximum and minimum values of the von Mises stress.
5. Use the Goodman alternating and mean stress interaction curve and S-N curve to estimate the number of cycles to failure. Use the reversed bending endurance limit.

Complex Loads



A part is subjected to completely reversed stresses as follows

σ_1 for n_1 cycles,

σ_2 for n_2 cycles,

σ_3 for n_3 cycles,

⋮

σ_m for n_m cycles,

What is the cumulative effect of these different load cycles?

Minor's Rule

Cumulative Damage Law

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \ldots + \frac{n_m}{N_m} = C$$

$n_i \equiv \text{number of cycles for stress level } i$

$N_i \equiv \text{cycles to failure at stress level } i$

$C \equiv \text{Constant ranging from } 0.7 \text{ to } 2.2.$

C is usually taken as 1.0

Minor's Rule is the simplest and most widely used Cumulative Damage Law

Example

Stress State	Cycles (n)	Life (N)	n/N
1	1,000	2,000	0.5
2	5,000	10,000	0.5
3	10,000	100,000	0.1
			1.1

Part will fail

Assignment
(Problem No. 1)

A rotating shaft is made of 42 x 4 mm AISI 1020 cold-drawn steel tubing and has a 6-mm diameter hole drilled transversely through it. Estimate the factor of safety guarding against fatigue failure when the shaft is subjected to a completely reversed torque of 120 N-m in phase with a completely reversed bending moment of 150 N-m. Use the stress concentration factor tables found in the appendices, and estimate the Marin factors using information in the body of the text.

Assignment
(Problem No. 2)

A solid circular bar with a 5/8 inch diameter is subjected to a reversed bending moment of 1200 in-lb for 2000 cycles, 1000 in-lb for 100,000 cycles and 900 in-lb for 10,000 cycles. Use the S-N curve used in this lecture. Determine whether the bar will fail due to fatigue. Assume all Marin factors are equal to 1.0.

Assignment
(Problem No. 3)

Same as Problem No. 2 except there is a constant axial force of 5,000 lb acting on the bar in addition to the completely reversed bending moment.