Introduction to Feynamn Diagrams

In order to construct Feynman Diagrams, one must know the elementary processes of each dynamical theory (QED, QCD and QFD).

Any combination of those represent possible reactions. The converse is not necessarily true: ere are things that do not have a proper Feynman diagram. We shall not worry about this here.

We're only interested in first order diagrams (no glueballs).

Conserved quantities in vertexes

Charge, lepton numbers and the baryon number are conserved in all vertexes.

Isospin, parity, charge conjugation and flavor are all conserved in QCD vertexes.

Energy and momentum, though conserved in all possible processes, need *not* be conserved in all vertexes.

QED

The elementary process is described by the following Feynman Diagram:[1]

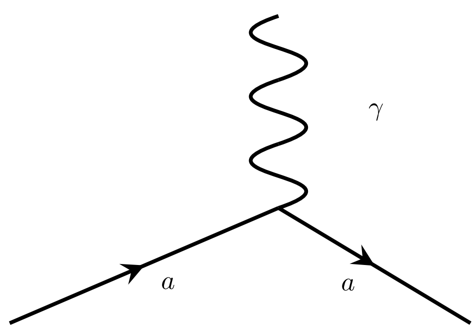

Figure 1: QED general primitive vertex

With a any charged particle (it must be charged, otherwise it won't interact through the EM force). Thus, a can be any quark, charged lepton (no neutrinos allowed), charged baryon or meson.

Notice that the particle coming out of the vertex is identical to the one coming in (both are "a"). If an up-quark comes in, an up-quark comes out, for instance.

All possible QED processes are combinations of variants of vertexes of this type. Conversely, all combinations of variants of vertexes of this type represent possible QED processes. By variants, I mean rotations of these vertexes (or,

[1] All diagrams will have time flowing from left to right.

actually, rotations of the lines around the vertexes): all configurations with one particle (one a or a photon) coming into the vertex and two coming out (a+photon or $a+a$) or one particle (one a or a photon) coming out of the vertex and two coming in (a+photon or $a+a$) are fine.

Notice that, if you rotate 1 by 180 degrees, the fermion arrows will point backwards in time. when this happens, *you must replace your fermion by its anti-particle.* For instance, if a was initially an electron, it will now be a positron.

QCD

In this case, only particles with color can interact. Only quarks have color, thus only quarks can interact through the strong force. The elementary vertex is:

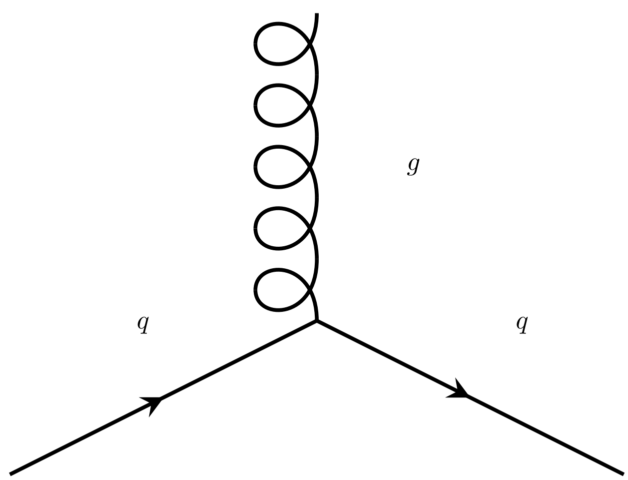

Figure 2: QCD general primitive vertex

Notice that the quarks entering and leaving the vertex are identical flavor-wise. If a down-quark enters, a down quark leaves. However, *they may have different color*. Color is conserved at each vertex, but the gluon carries two color charges, and so it is possible for the colors of the quarks to differ. For example, you can have a red strange quark $s(r)$ coming in and a blue strange quark $s(b)$ coming out. In this case, the gluon will be blue and anti-red: $g = g(b, \bar{r})$. Notice that the net color is blue both before and after the vertex.

Just like in QED, all possible QCD processes are combinations of variants of vertexes of this type. Conversely, all combinations of variants of vertexes of this type represent possible QCD processes.

QFD

The weak force acts on everything with flavor (hence the name quantum flavor dynamics). We thus have two types of interactions: lepton-lepton and quark-quark (note that we can't have vertexes of the type lepton-quark, since the lepton numbers must be conserved in any vertex).

Leptons

There are charged and neutral interactions, mediated by the bosons W^+(or W^-) and Z^0, respectively.

Charged interaction

If we have a non-neutrino lepton (charge -1) entering a vertex of a *charged* interaction, the only way to have both the lepton number and charge conserved is to have the *negatively* charged boson W^- as the mediator and to have the neutrino which corresponds to our lepton leaving the vertex.

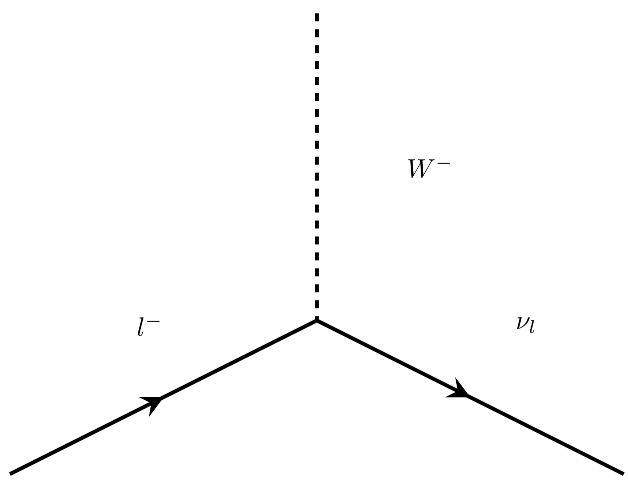

Figure 3: QFD negatively charged lepton-lepton primitive vertex

If we have a non-anti-neutrino anti-lepton (charge +1) entering a vertex of a *charged* interaction, the only way to have both the lepton number and charge conserved is to have the *positively* charged boson W^+ as the mediator and to have the anti-neutrino which corresponds to our anti-lepton leaving the vertex.

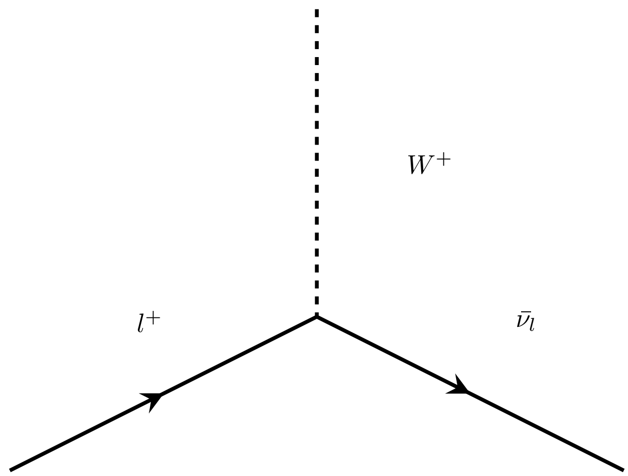

Figure 4: QFD positively charged lepton-lepton primitive vertex

Neutral interaction

If we have a non-neutrino lepton (charge -1) entering a vertex of a *neutral* interaction (mediated by the boson Z^-), the only way to have both the lepton number and charge conserved is to have an identical lepton (*i.e.* same flavor) leaving the vertex.

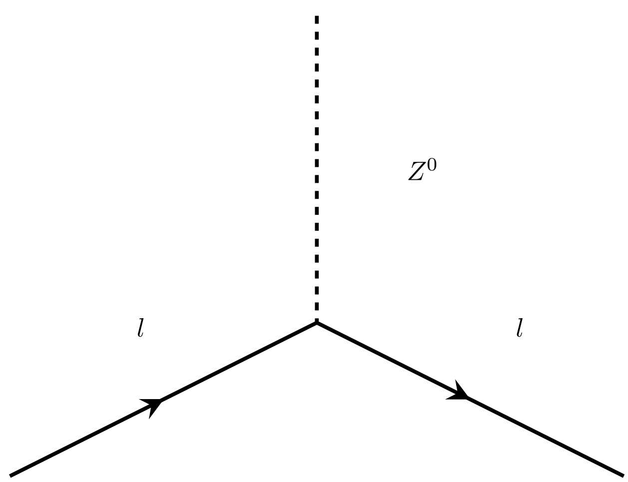

Figure 5: QFD neutral lepton-lepton primitive vertex

Quarks

The flavor can change in weak processes! Most probably, it changes without changing the generation. In that case, the six quarks can be transformed in one another in the following way:

$$\begin{array}{ccc} u & \leftrightarrow & d \\ c & \leftrightarrow & s \\ t & \leftrightarrow & b \end{array}$$

However, these are not the only options. In general, we have (we may see this as an experimental fact):

$$\begin{array}{ccc} u & \leftrightarrow & d\text{'} \\ c & \leftrightarrow & s' \\ t & \leftrightarrow & b' \end{array} \tag{1}$$

with

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix} \tag{2}$$

The matrix above is called the Cabbibo-Kobayashi-Maskawa matrix V_{CKM}.

The diagonal elements of this matrix have (experimentally obtained) values of about 0.970 to 0.999, while the others range from 0 to 0.25.

Hence, when we have an up-quark changing flavor, we get

$$u \to d' = V_{ud}d + V_{us}s + V_{ub}b$$

and the probability of getting $u \to d$ is approximately $|V_{ud}|^2 \sim 0.95$ while the probabilities of getting $u \to s$ and $u \to b$ are $|V_{us}|^2 \sim 0.04$ and $|V_{ud}|^2 \sim 0.01$, respectively.

That said, let's look at the vertexes themselves.

Charged interaction

If we have a negatively charged quark (charge -1/3) entering a vertex of a *charged* interaction, the only way to have both the lepton numbers (0) and charge conserved is to have the *negatively* charged boson W^- as the mediator and to have a positively charged quark (charge $+2/3$) leaving the vertex.

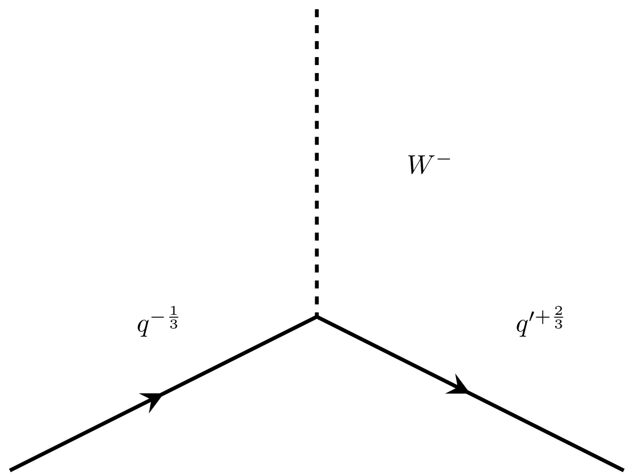

Figure 6: QFD negatively charged quark-quark primitive vertex

If we have a positively charged quark (charge $+2/3$) entering a vertex of a *charged* interaction, the only way to have both the lepton numbers (0) and charge conserved is to have the *positively* charged boson W^+ as the mediator and to have a negatively charged quark (charge -1/3) leaving the vertex.

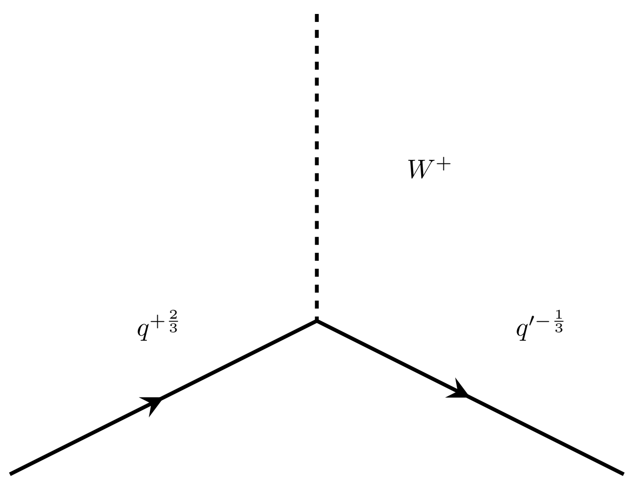

Figure 7: QFD positively charged quark-quark primitive vertex

The interactions involving anti-quarks come naturally from variations of these vertexes, just as in the lepton's case.

In fact, this last vertex (7) is unnecessary. We can obtain it by rotating 6 by $\boxed{\text{???how can we obtain it from the other one???}}$

Notice that these charged interactions can "change" a quark's flavor from one of the positively charged quarks to one of the negatively charged quarks, or vice-versa, just like we described in 1 and 2.

Neutral interaction

If we have a quark (charge -1/3 or +2/3) entering a vertex of a *neutral* interaction (mediated by the boson Z^0), the only way to have both the lepton numbers (0) and charge conserved is to have a quark with the same charge as our initial quark coming out of the vertex. This is not sufficient to forbid changes of flavor in neutral currents! (It could be up-quark to charm-quark, for instance). However, experimentally we verify that it just doesn't happen.

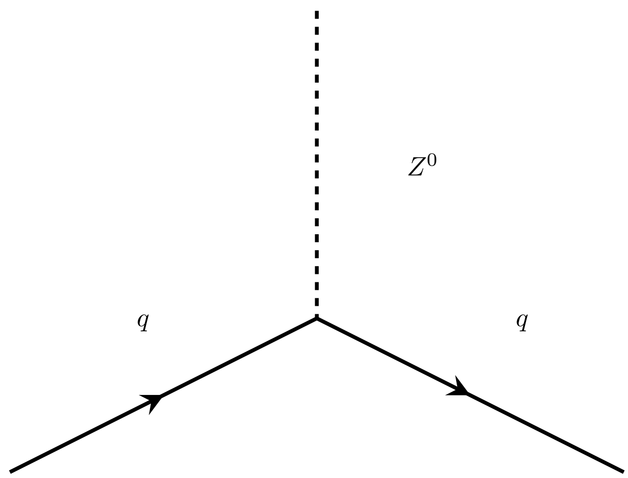

Figure 8: QFD neutral quark-quark primitive vertex

Note that the quarks coming in and out are indeed identical.

Complete list of all QED vertexes

As we've seen above, all possible QED vertexes can be obtained by rotating 1 in the prescribed way. For completeness, we shall list here all the resulting vertexes. Some comments about the construction of these vertexes will be made.

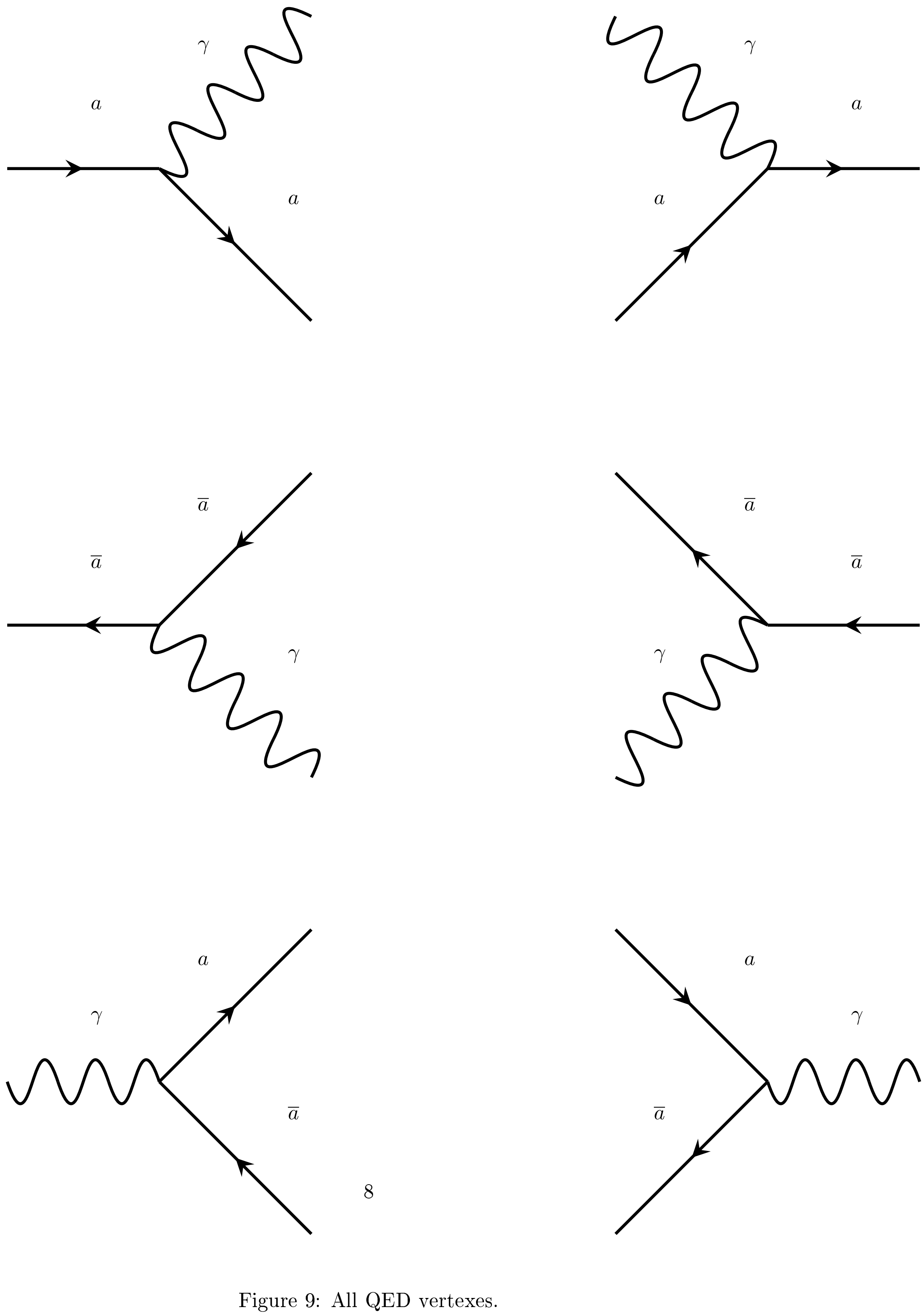

Figure 9: All QED vertexes.

In order to figure out what are the possible vertexes, one can think of all the possibilities for the incoming side of the vertex (the left) and then see what particles appear on the right when one rotates the elementary vertex.

For one particle coming in, we obviously have 3 cases: a particle a comes in; a particle $\bar{a}$ comes in; a photon comes in. The particles coming out of the vertex are given by rotations of the elementary vertex 1 in such a way that the incoming particle is the desired one. The result is (1,1), (2,1) and (3,1)[2].

The other possible case is to have two particles coming in. The possible pairs of incoming particles are (a,a),$(a,\bar{a})$,$(\bar{a},\bar{a})$,(γ,a) and $(\gamma,\bar{a})$. However, there are no rotations of 1that lead to two particles a or two particles $\bar{a}$ on the lhs (left-hand-side). Therefore, only the pairs $(a,\bar{a})$, (γ,a) and $(\gamma,\bar{a})$ are possible pairs of incoming particles. As we shall see, this process of "finding out the possible pairs of incoming particles and weeding out the impossible ones" is fortunately unnecessary. In fact, we can focus only on the 1-incoming particle cases and then perform a reflection on the vertical line which passes through the vertex (while it is true that reflections are not proper rotations, it's easy to see why they can be accepted as proper rotations in these diagrams: since up and down have no meaning in Feynman Diagrams, the 180º rotation about the vertex of (2,1) gives a diagram equivalent to (2,2), for instance).

Let's say a few words about each of the diagrams of 9:

- (1,1) is the elementary vertex we've already discussed above. The photon is tilted so that it is clear that it comes out of the vertex.
- (2,1) is obtained from (1,1) by rotating the incoming a to the place of the photon, the photon to the place of the out-coming a, and this latter to the place of the incoming a. We then replace a's with $\bar{a}$'s where the arrows are reversed.
- (3,1) is obtained from (2,1) just like (2,1) is obtained from (1,1).
- (1,2) is obtained from (1,1) by a reflection.
- (2,2) is obtained from (1,2) just like (2,1) is obtained from (1,1) or from (2,1) by a reflection.
- (3,2) is obtained from (2,2) just like (2,1) is obtained from (1,1), or from (3,1) by a reflection.

[2] I'll refer to the figures in 9 by the pair (row, column). For example, the figure (3,1) is the $\gamma \to \bar{a} + a$ vertex.