

solve the functional equation:  $f(\sqrt{x^2+y^2})=f(x)f(y)$   
solution:

$$f(x)=f\left(\sqrt{\left(\frac{x}{\sqrt{2}}\right)^2+\left(\frac{x}{\sqrt{2}}\right)^2}\right)=f\left(\frac{x}{\sqrt{2}}\right)^2$$

repeating we obtain

$$[f(x)]^{\left(\frac{1}{2^n}\right)}=f\left(\frac{x}{2^{\left(\sum_{k=1}^n \left(\frac{1}{2^k}\right)\right)}}\right)$$

at  $n$  tends  $\infty$

$$1=f\left(\frac{x}{2}\right)$$

therefore  $f(x)=1$  is a solution