

Method for finding Flow Rate for compressing gas into a fixed volume

$$\text{mass}_{\text{dot.in}} = \text{mass}_{\text{dot.out}} \quad \text{Anything with .dot in the name is wrt time}$$

$$V_{\text{dot}} \cdot \rho = \frac{d}{dt}(V\rho) \quad \text{where } V \text{ (volume) and } \rho \text{ (density)}$$

$$V_{\text{dot}} \cdot \rho = V \cdot \left( \frac{d}{dt} \rho \right) + \rho \cdot \left( \frac{d}{dt} V \right)$$

$$V_{\text{dot}} \cdot \rho = V \cdot \left( \frac{d}{dt} \rho \right) \quad \text{since } \left( \frac{d}{dt} V \right) = 0 \text{ Volume does not change}$$

$$V_{\text{dot}} \cdot \left( \frac{M \cdot P}{R \cdot T} \right) = V \cdot \left( \frac{M}{R \cdot T} \right) \quad \text{since } \left( \rho = \frac{M \cdot P}{R \cdot T} \right) \text{ and } \left[ \frac{d}{dt} (\rho) = \frac{M}{R \cdot T} \right], \text{ and } \left( \frac{d}{dt} P \right) = V_{\text{dot}} \cdot \frac{P}{V}$$

$$\frac{V_{\text{dot}}}{V} \cdot \int_{t_0}^t \frac{1}{P} dP = \int_{P_0}^P \frac{1}{P} dP \quad \text{rearrange and integrate}$$

$$\frac{V_{\text{dot}} \cdot t}{V} = \ln \left( \frac{P}{P_0} \right) \quad \text{answer}$$

$$V_{\text{dot}} = \frac{V \cdot \ln \left( \frac{P}{P_0} \right)}{t} \quad \text{after rearranging terms, flow rate in terms of time}$$