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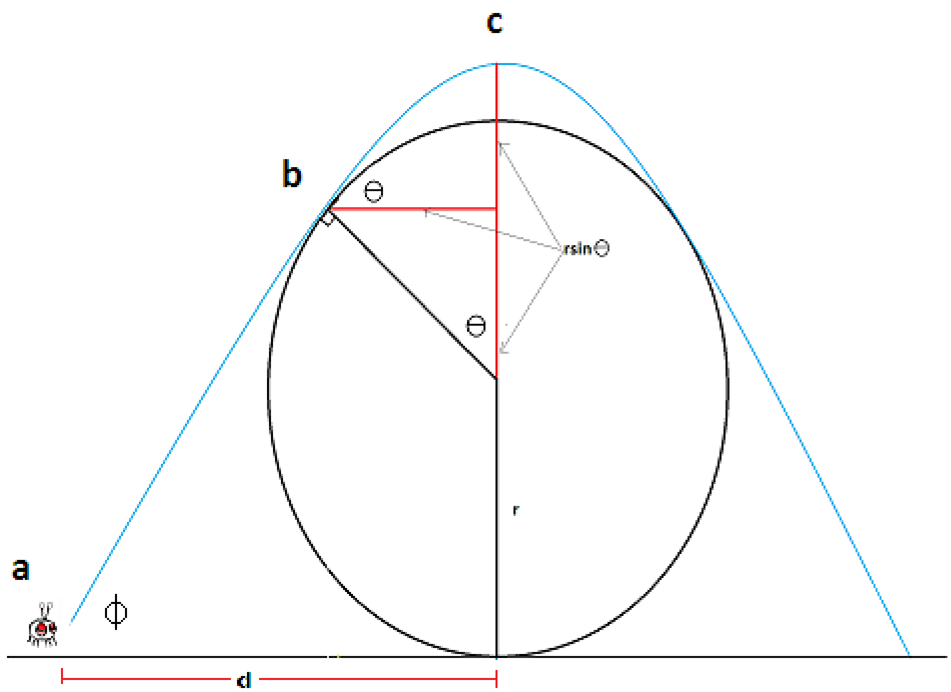
Extra Credit
Physics4a

A lazy flea approaches a stationary log with radius R. At what distance from the log must the flea be to jump over the log with the least amount of effort.



a, b: $\frac{1}{2}mv_a^2 = \frac{1}{2}mv_b^2 + mg(r + rcos\theta) \to \boxed{v_a^2 = v_b^2 + 2gr + 2grcos\theta}$

b, c: $x = v_0 t \to rsin\theta = v_b cos\theta t \to \boxed{t = \frac{rsin\theta}{v_b cos\theta}}$

$v = v_0 + at \to 0 = v_b sin\theta - gt \to \boxed{v_b = \frac{gt}{sin\theta}} \to \boxed{v_b^2 = \frac{gr}{cos\theta}} \to \boxed{v_a^2 = \frac{gr}{cos\theta} + 2gr + 2grcos\theta}$

$\frac{dv_a}{d\theta} = \frac{grsec\theta tan\theta - 2grsin\theta}{2v_a} \to \frac{grsin\theta - 2grsin\theta cos^2\theta}{2v_a cos^2\theta}$ $\boxed{\theta = 45^0}$

$\frac{dv_a}{d\theta} = 0$ when $grsin\theta(1 - 2cos^2\theta) = 0$ so v_a is minimum when $cos\theta = \frac{1}{\sqrt{2}} \uparrow$

$\to \boxed{v_{a_{min}}^2 = \frac{gr}{\sqrt{2}} + 2gr + \frac{2gr}{\sqrt{2}}}$

$v_b = \sqrt{\frac{gr}{cos\theta}} \quad v_{b_x} = v_{a_x} = v_c = \sqrt{grcos\theta}$

c, a: $v^2 = v_o^2 + 2a(x - x_0) \to d = \frac{v_a^2 - v_c^2}{2g} \to \boxed{gr(sec\theta + 2 + cos\theta)}$

$v_{a_y}^2 = v_{a_x}^2 + v_a^2 \to \boxed{v_{a_y}^2 = \frac{gr}{cos\theta} + 2gr + grcos\theta}$

$tan\phi = \frac{v_{a_y}}{v_{a_x}} \to arctan\frac{sec\theta + 2 + cos\theta}{cos\theta} \to \boxed{\phi = 67.5^0}$