Pauli-Villars regularization is still used for many calculations, but another method has become more popular because it works better for non-Abelian gauge fields: the *dimensional regularization* of 't Hooft and Veltman [**118**]. The idea of dimensional regularization is to perform an analytic continuation in the number d of space-time dimensions. This is not as crazy as it sounds at first! We are not proposing to develop a theory of integration in d dimensions for arbitrary complex d, only to extend certain special kinds of d-dimensional integrals to complex d. Nor are we claiming that there is only one way to do so; all we need is one way that works.

To apply this method to a divergent integral of the form (7.1), one prepares the ground by retracing the steps we have used to evaluate *convergent* integrals:

i. Wick-rotate the momenta.
ii. Convert the denominator from a product of quadratics to a power of a single quadratic by Feynman's formula, obtaining an integral of the form (7.7), and then interchange the momentum space integration with the integration over the Feynman parameters.
iii. Perform a linear change of variable to get rid of the linear terms in the denominator and obtain an integral of the form (7.8).
iv. Reduce the resulting integral to integrals of radial functions on $\mathbb{R}^{4L}$ by using the formula (7.10).

In the present setting, each of these steps consists of *formal* manipulations. They must be regarded simply as parts of a symbolic calculation that will eventually interpret the original divergent integral as a limit of well-defined finite quantities.

It is with the final reduction to integrals of radial functions that we can start doing some honest analysis. Specifically, for a radial function on $\mathbb{R}^d$, say $f(|q|)$, integration in polar coordinates reduces its d-dimensional integral to a one-dimensional one:

$$\int f(|q|)\, d^d q = \Omega_d \int_0^\infty f(r) r^{d-1}\, dr, \tag{7.13}$$

where $\Omega_d = 2\pi^{d/2}/\Gamma(d/2)$ is the area of the unit sphere in $\mathbb{R}^d$. (See, e.g., Folland [**48**].) Now, the expression on the right *does* define an analytic function of d in the domain of those d for which the integral converges. For our purposes $f(r)$ will be of the form $f(r) = r^{2k}/(r^2+c^2)^n$, so the integral converges provided $0 < d < 2n-k$ and can be evaluated in terms of the gamma function: the substitution $t = (r/c)^2$

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$$\begin{aligned}
\int_0^\infty \frac{r^{2k+d-1}}{(r^2+c^2)^n}\, dr &= \frac{c^{2k+d-2n}}{2} \int_0^\infty \frac{t^{k+(d/2)-1}\, dt}{(1+t)^{-n}} \\
&= \frac{c^{2k+d-2n}}{2} B(k+\tfrac{1}{2}d,\, n-k-\tfrac{1}{2}d) \\
&= \frac{c^{2k+d-2n}}{2} \frac{\Gamma(k+\frac{1}{2}d)\Gamma(n-k-\frac{1}{2}d)}{\Gamma(n)}.
\end{aligned} \tag{7.14}$$

This expression continues analytically to larger values of d except for poles where $n-k-\frac{1}{2}d$ is a nonnegative integer. In our situation, one of these poles will occur at $d=4$, which corresponds to the original divergent integral (when we integrate over the loop momenta one at a time). The "∞" of the original integral then becomes a "$1/(4-d)$" term in a well-defined analytic expression that can be used in further calculations.

The shortcut that we discussed for evaluating convergent integrals after (7.8)