

You are allowed ten minutes before the start of the examination to acquaint yourself with the instructions below and to read the question paper.

Do not write anything until the invigilator informs you that you may start the examination. You will be given five minutes at the end of the examination to complete the front of any answer books used.

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June 2003

X3PHLJ 2002/3 A 001  
1 Answer Book  
All types of calculator permitted  
providing they cannot store text

## **THE UNIVERSITY OF READING**

Part III Examination in Science

Physics Unit 3/PH/LJ

**FRACTALS AND CHAOS**

1½ Hours

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Answer TWO Questions

1. The generalised dimension  $D_q$  of a fractal object is found from plotting

$$\log \left| \sum_i p_i^q \right| = \log \left| \left\langle C_i^{q-1}(\varepsilon) \right\rangle \right|$$

against  $\log(\varepsilon)$  to yield  $(q-1)D_q$  from the gradient. Recall also

$$C_i(\varepsilon) = \frac{1}{N} \sum_{j=1}^N H\left(\varepsilon - |r_i - r_j|\right)$$

and

$$H(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

- (a) Define the terms in the above expressions. What significance do the values of  $q=0$  and  $q=2$  have?

[6 marks]

- (b) A fractal object has the following set of x-coordinates listed below in rank order. Calculate a suitable correlation function for these points, and hence calculate the box-counting and correlation dimensions for the fractal. Give a detailed description of your working.

[14 marks]

1	2	3	4	5	6	7	8
0.0021	0.0103	0.0267	0.0350	0.0761	0.0844	0.1008	0.1091

  

9	10	11	12	13	14	15	16
0.2243	0.2325	0.2490	0.2572	0.2984	0.3066	0.3230	0.3313

  

17	18	19	20	21	22	23	24
0.6687	0.6770	0.6934	0.7016	0.7428	0.7510	0.7675	0.7757

  

25	26	27	28	29	30	31	32
0.8909	0.8992	0.9156	0.9239	0.9650	0.9733	0.9897	0.9979

2. The Tent Map is defined as

$$T_A(x) = \begin{cases} 2Ax, & 0 \leq x \leq 0.5 \\ 2A(1-x), & 0.5 < x \leq 1 \end{cases}$$

- (a) Sketch the Tent Map with amplitude  $A=0.8$  and illustrate iterations on your diagram starting from  $x=0.8$ .

[8 marks]

- (b) Describe what is meant by fixed-points, stability and the Lyapunov exponent. Indicate fixed points on your diagram from part (a) and discuss their stability.  
[6 marks]
- (c) Show that the long-term orbits of the Tent Map are localised within the interval  $[2A(1-A), A]$ .  
[6 marks]
3. Discuss techniques for controlling chaos, using sketches of the Logistic Map to illustrate the ideas. Describe some experimental systems where the ideas have been successfully implemented.  
[20 marks]

(End of Question Paper)