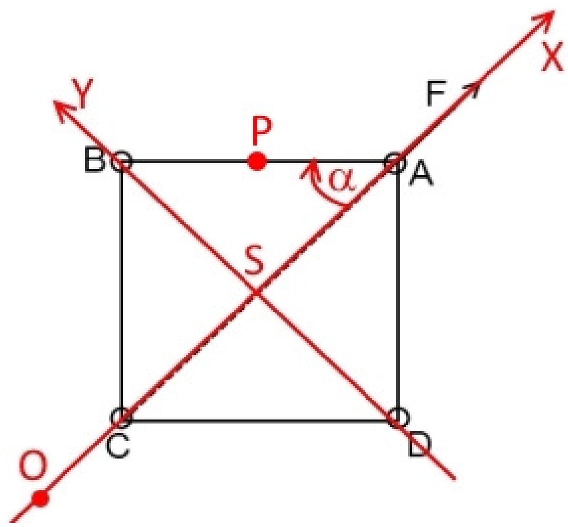

Problem. *There are four homogeneous rods. Each rod has a mass m and a length b. The ends of the rods are connected by frictionless hinges such that the rods form a rhombic frame (see the picture). This frame is shaped as square $ABCD$ and put at rest on a smooth horizontal table. Then one applies a force F to the hinge A along the diagonal AC. Find acceleration of the point C right after the force has been applied.*

The result is
$$a_C = -\frac{F}{8m}.$$

Road map of the solution. Let SXY be the König reference frame. (It moves together with the center of mass and does not rotate.) A point O is fixed relative the lab frame; $\boldsymbol{F} = F\boldsymbol{e}_x$.

Introduce generalized coordinates α, x such that $\boldsymbol{OS} = x\boldsymbol{e}_x$, for the angle α see the picture.

By the König theorem the kinetic energy of the system is
$$T = \frac{1}{2}(4m)\dot{x}^2 + T_*;$$
here
$$T_* = 4\Big(\frac{1}{2}m\Big|\frac{d}{dt}\boldsymbol{SP}\Big|^2 + \frac{1}{2}J_P\dot{\alpha}^2\Big)$$
is the kinetic energy of the system relative the König frame;
$$\boldsymbol{SP} = \frac{b}{2}(\cos\alpha\boldsymbol{e}_x + \sin\alpha\boldsymbol{e}_y), \quad J_P = mb^2/12.$$
The generalized forces are
$$Q_x = \Big(\frac{\partial \boldsymbol{OA}}{\partial x}, \boldsymbol{F}\Big), \quad Q_\alpha = \Big(\frac{\partial \boldsymbol{OA}}{\partial \alpha}, \boldsymbol{F}\Big),$$

here

$$\boldsymbol{OA} = (x + b\cos\alpha)\boldsymbol{e}_x.$$

The equations of motion are

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{x}} - \frac{\partial T}{\partial x} = Q_x, \quad \frac{d}{dt}\frac{\partial T}{\partial \dot{\alpha}} - \frac{\partial T}{\partial \alpha} = Q_\alpha.$$

The initial conditions are

$$\alpha(0) = \pi/4, \quad \dot{\alpha}(0) = 0, \quad \dot{x}(0) = 0.$$

The acceleration is expressed as follows

$$\boldsymbol{a}_C = \frac{d^2}{dt^2}\boldsymbol{OC}, \quad \boldsymbol{OC} = (x - b\cos\alpha)\boldsymbol{e}_x.$$

The result is

$$\boldsymbol{a}_C = -\frac{F}{8m}\boldsymbol{e}_x.$$

There is also an elementary way to solve this problem: replace the Lagrange equations with:

$$4m\frac{d^2}{dt^2}\boldsymbol{OS} = \boldsymbol{F}, \quad \dot{T}_* = \Big(\frac{d}{dt}\boldsymbol{SA}, \boldsymbol{F}\Big).$$