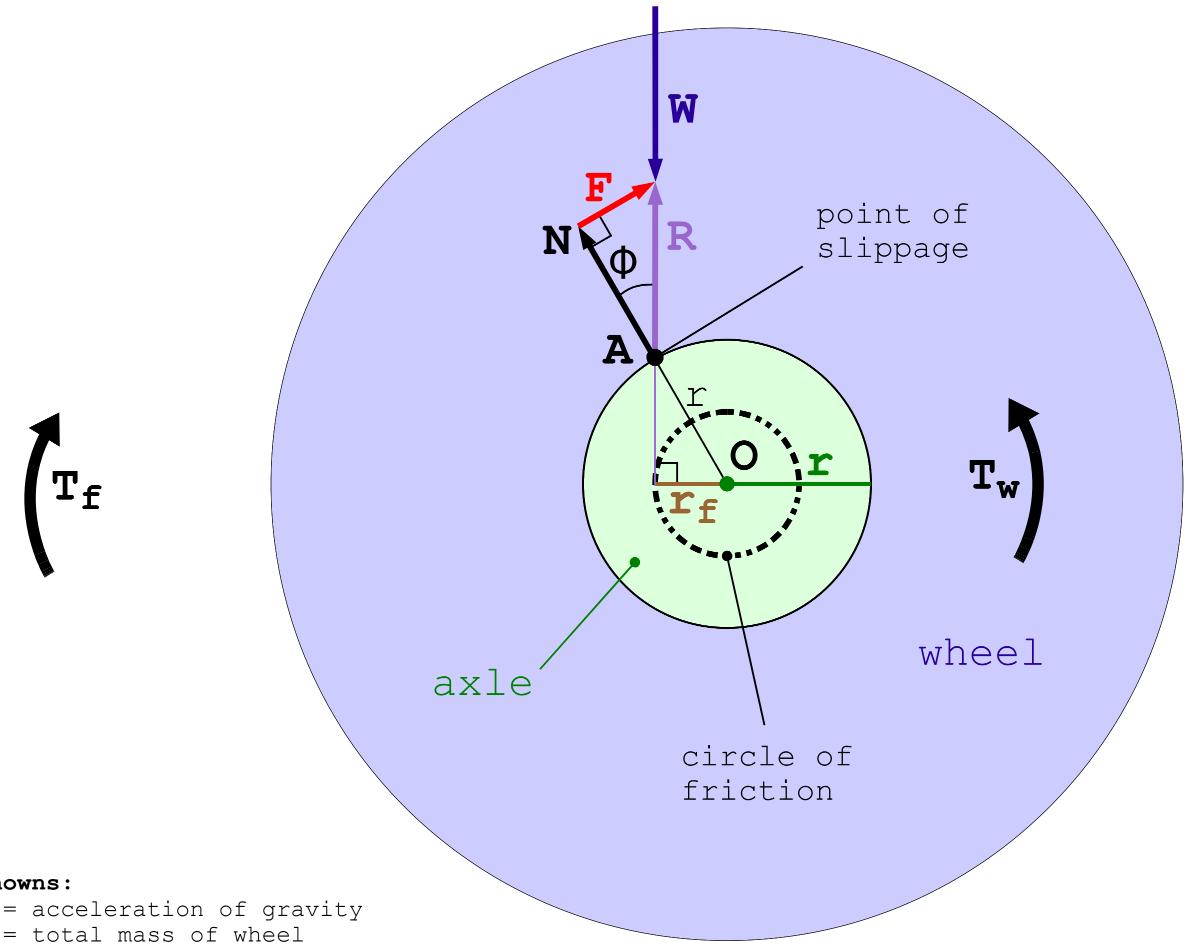
Frictional Torque
(Axle Friction)



Knowns:
G = acceleration of gravity
m = total mass of wheel
r = radius of axle ≈ inner radius of wheel
μ_k = coefficient of kinetic friction
W = mG = force of wheel's weight

Unknown:
N = normal force, perpendicular to contact of wheel to axle
F = tangential force of friction from wheel against axle (**axle friction**)
ϕ = angle of friction
R = reactionary force to the wheel's weight and rotation
r_f = radius of circle of friction
T_f = rF = opposing torque due to friction (**frictional torque**)
T_w = r_fW = minimum torque required to keep wheel's rotation at constant speed
O = center of axle (and wheel)
A = point of slippage, where the wheel contacts the axle

Given:
$\mu_k = \tan\phi \approx \sin\phi$ for small values of ϕ (for < 45°)

Solve:
Calculate the magnitude of T_w, the minimum torque necessary to maintain the wheel's angular velocity at a constant speed.

Solution:
Write equilibrium equations that balance the vertical forces and the angular forces (torques). In this case, we can imagine equilibrium as summing to zero, as T_w should exactly cancel out frictional forces.
$+\uparrow\Sigma F_y = 0$: $R - W = 0$ $R = W = mG$ (sum of vertical forces)
$+\nwarrow\Sigma M_O = 0$: $T_w - T_f = 0$ $T_w = T_f$ (sum of torques about point O, the center)

Consider:
$F = R\sin\phi$ (tangential component of reaction force, R)
$F = W\sin\phi$ $\mu_k \approx \sin\phi$ $F \approx W\mu_k$
$T_f = rF$ (friction F applied with radius r at point A)
$T_f \approx rW\mu_k$

$$\mathbf{T_w = T_f \approx r\mu_k mG}$$